



Linear, Exponential and Sigmoid Growth

The Mathematical Models of Biological Growth Rates

The Meaning of Growth Rate

Growth rate is defined as the **increased growth per unit time**. It is the mathematical expression of an organism's biological expansion.

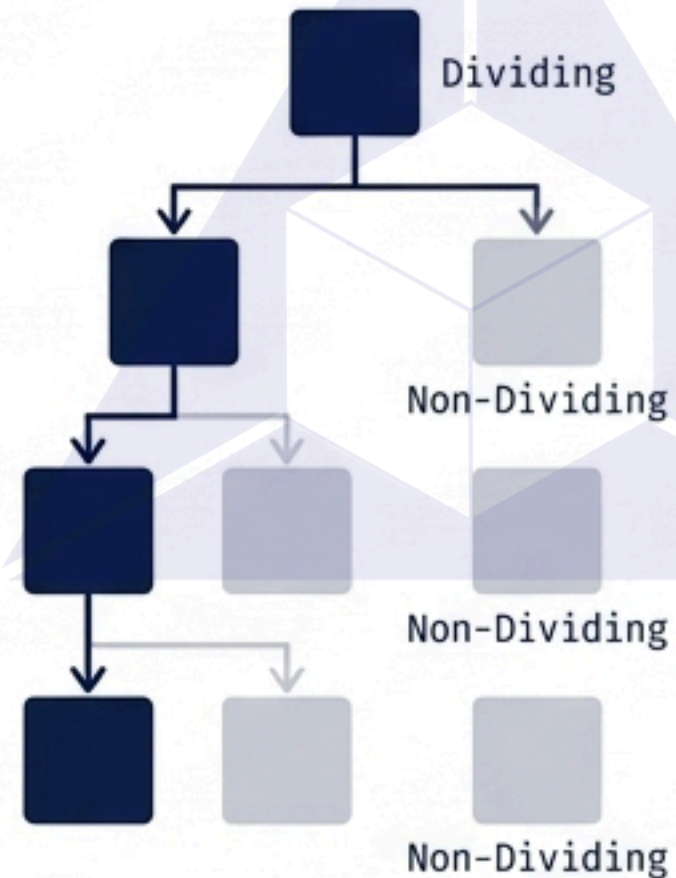
i Biological Context: Growth at a cellular level is driven by an increase in protoplasm. However, because protoplasm is hard to measure directly, we measure proportional proxies like fresh weight, dry weight, length, area, volume, or cell number.

An organism can produce more cells in a variety of ways, but mathematically, this growth generally follows one of two foundational patterns:
Arithmetic or Geometric.

The Mechanics of Cell Division

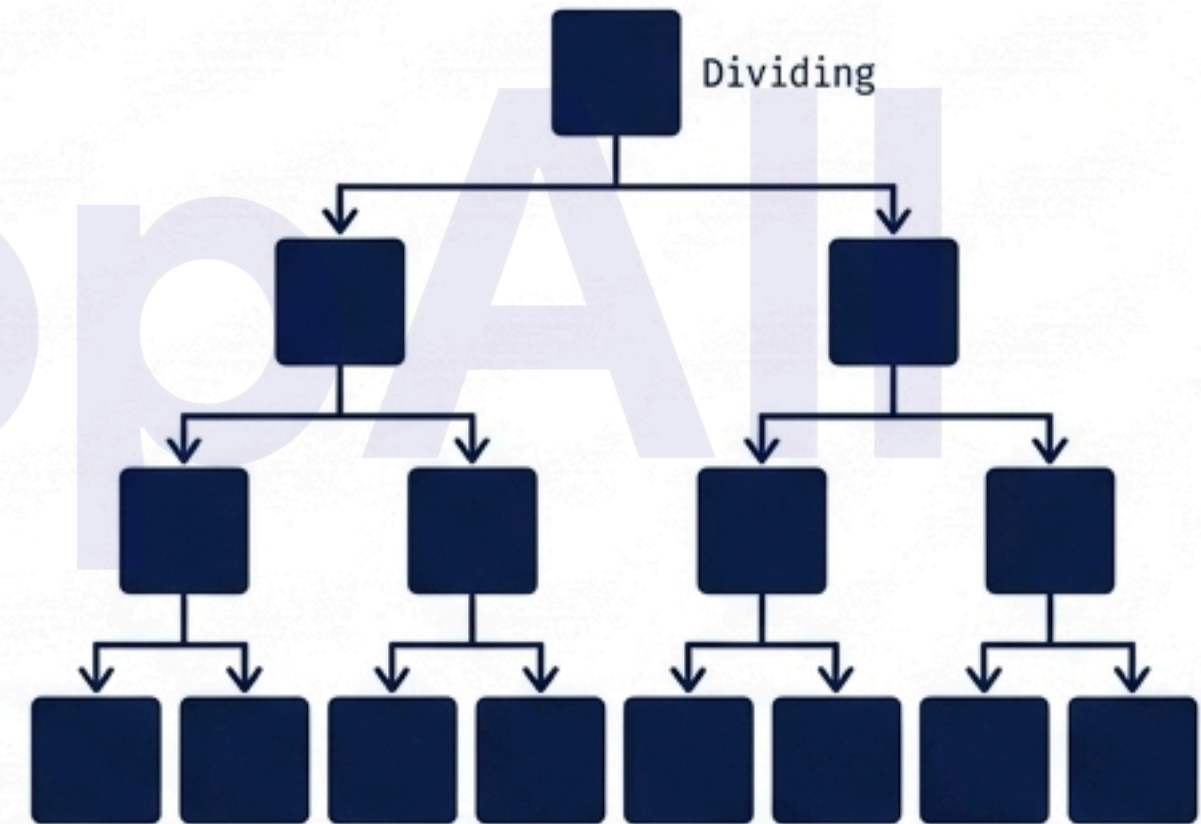
Figure 13.4: Arithmetic vs. Geometric

Arithmetic Phase



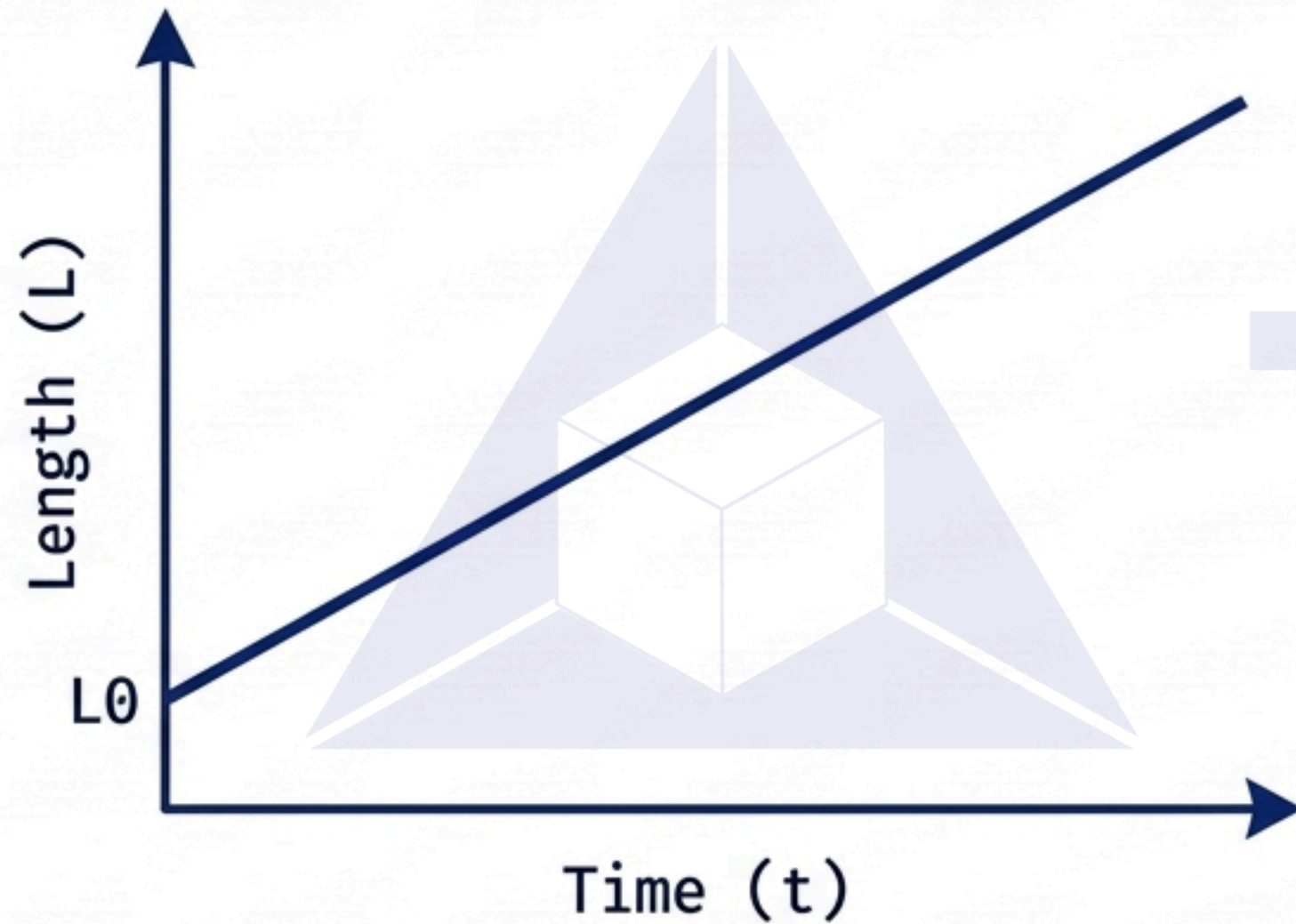
Mitotic division where only one daughter cell retains dividing capacity.

Geometric Phase



Mitotic division where *both* progeny cells retain the ability to divide.

Arithmetic Growth: The Steady March



Formula Teardown

$$L_t = L_0 + rt$$

length at time "t"

length at time "zero"

growth rate / elongation per unit time

The simplest biological expression of arithmetic growth is a plant root elongating at a constant, linear rate.



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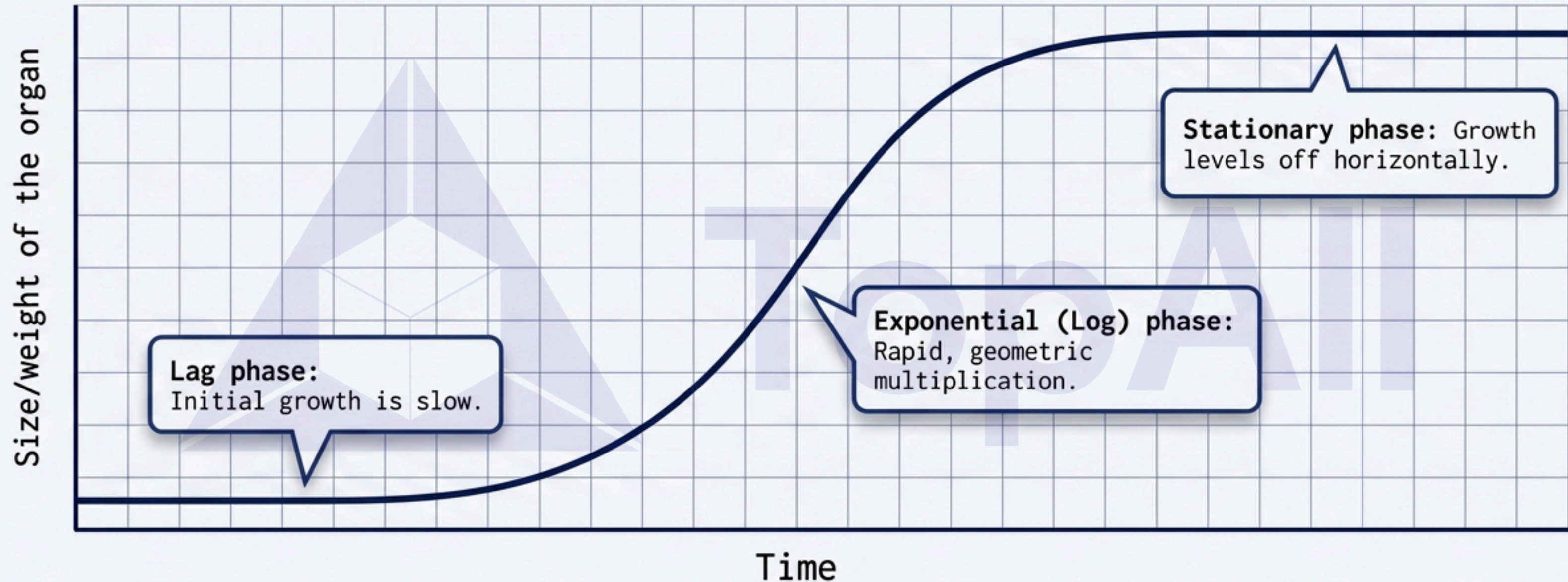
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Geometric Growth & The Sigmoid Curve

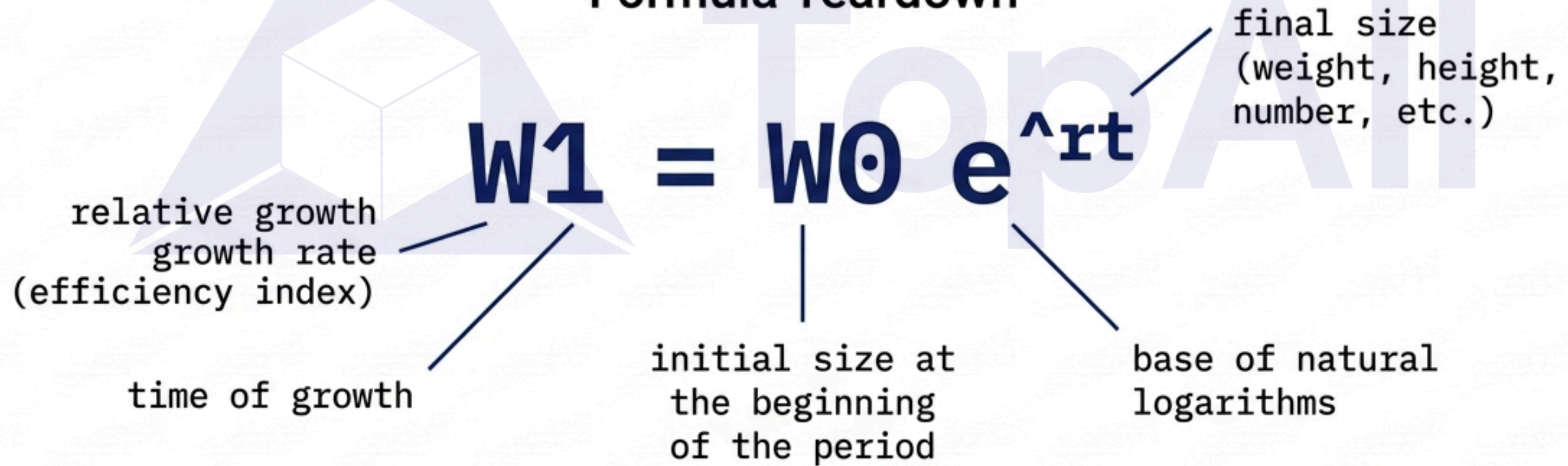


The Sigmoid (S-curve) is typical for living organisms, cells, tissues, and plant organs growing in a natural environment where limited nutrient supply eventually forces the stationary phase.

Modeling Exponential Growth

During the log phase, both progeny cells continue to divide, creating exponential geometric expansion.

Formula Teardown



The diagram illustrates the formula for exponential growth, $W_1 = W_0 e^{rt}$, with labels for each component:

- W_1 : final size (weight, height, number, etc.)
- W_0 : initial size at the beginning of the period
- e : base of natural logarithms
- r : relative growth growth rate (efficiency index)
- t : time of growth

Understanding 'r' (Relative Growth Rate)



In the geometric formula, the variable r represents the relative growth rate.

The Efficiency Index

r is a measure of the fundamental ability of the plant to produce new plant material.

The Snowball Effect

Because geometric growth is exponential, the final size (W_1) is highly dependent on the initial size (W_0).

Comparing Growth Models

Feature

Arithmetic Growth

Geometric Growth

Cell Division

One cell divides, one matures.

Both progeny cells continue to divide.

Graph Shape

Linear (Straight line).

Sigmoid (S-Curve).

Mathematical Model

$$L_t = L_0 + rt$$

$$W_1 = W_0 e^{rt}$$

Environment Context

Constant, unconstrained elongation rate.

Natural environment with eventual nutrient limitations.

Two Perspectives on Growth Measurement

Absolute Growth Rate

Measurement and comparison of total growth per unit time.

The Core Metric: "How much actual physical size was added?"

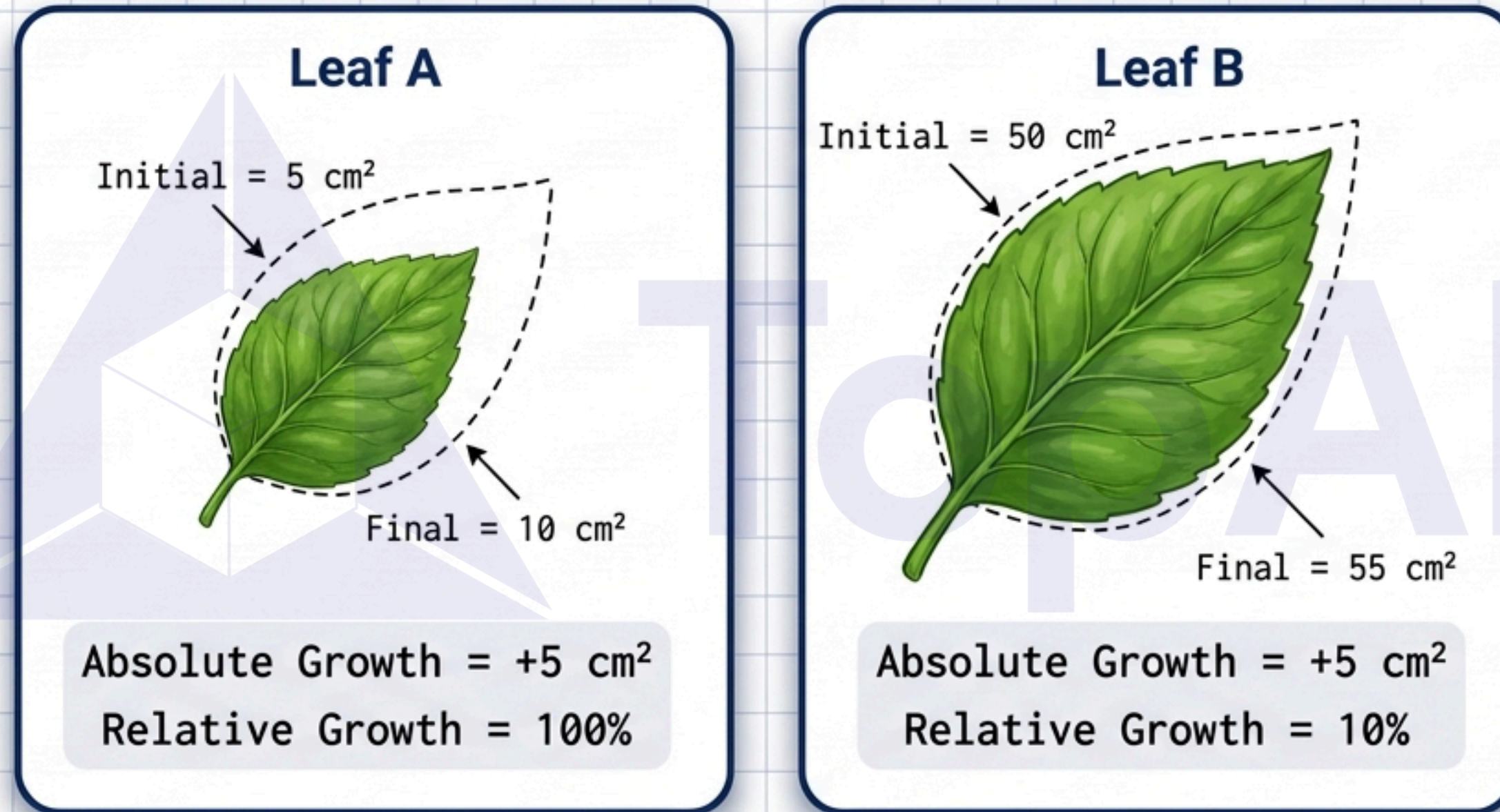
Relative Growth Rate

Growth of the given system per unit time expressed on a common basis (e.g., per unit initial parameter).

The Core Metric: "How much did it grow compared to its starting size?"

The Illusion of Absolute Growth

Figure 13.7: Comparative Growth Rates



Both leaves show the exact same **absolute growth** (+5 cm²). However, Leaf A exhibits a vastly higher **relative growth rate** because its identical physical gain represents a doubling of its smaller initial mass.



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