

Motion in a Plane · Concept 9

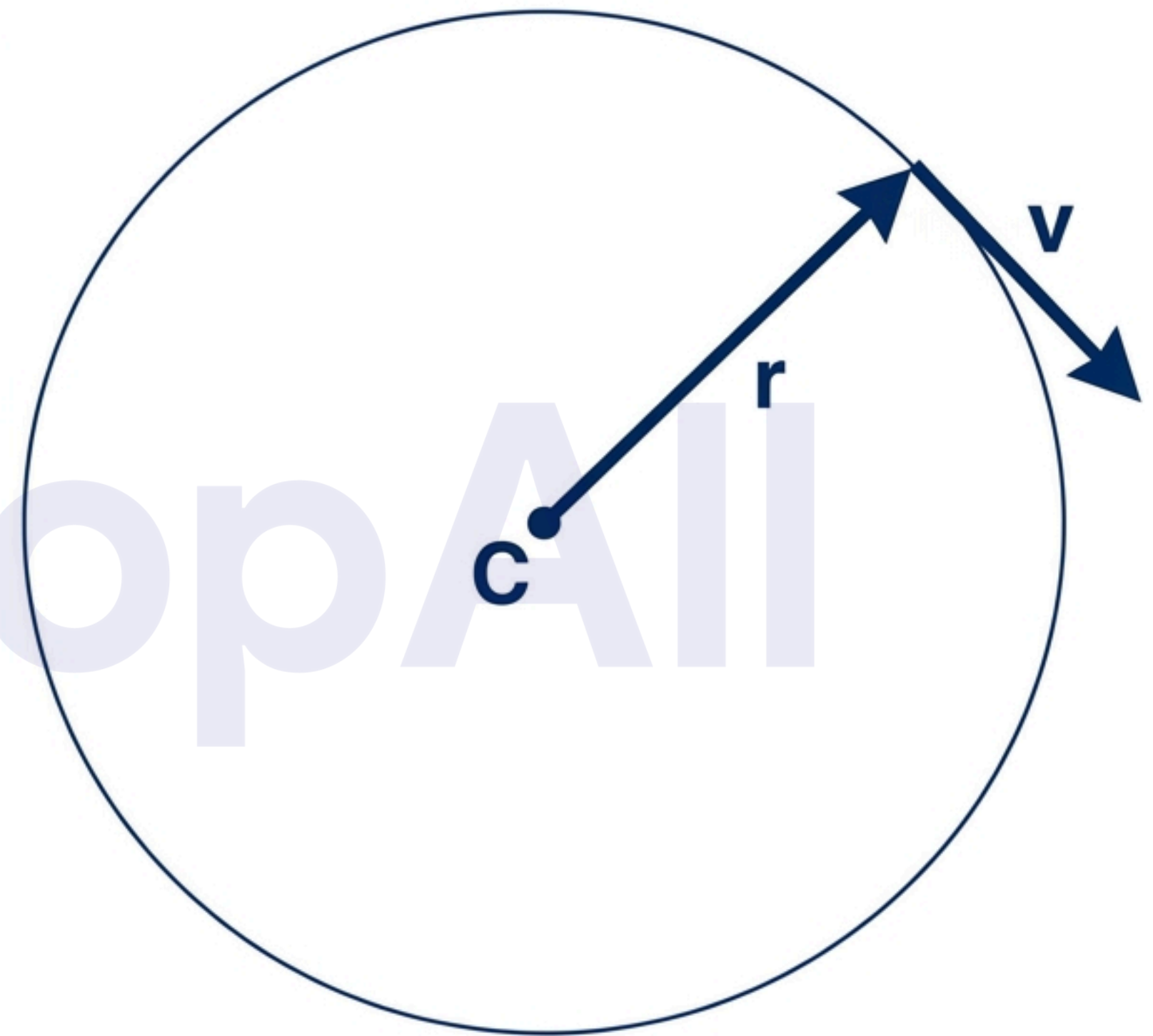
Uniform Circular Motion

Decoding the geometry, formulas, and kinematics of constant-speed circular paths.

The Constant Speed Paradox

An object follows a circular path at a uniform (constant) speed.

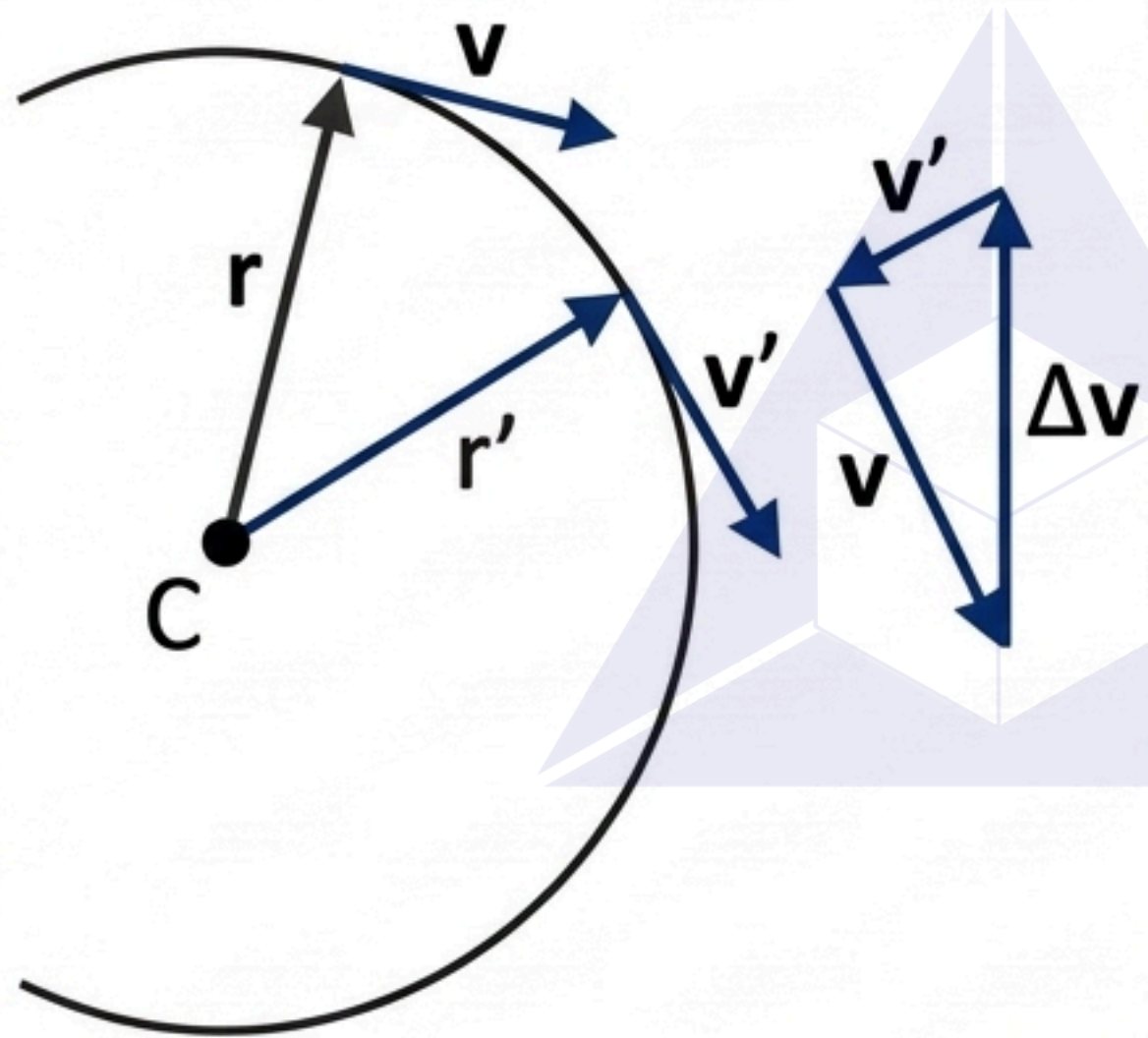
Because the velocity vector's direction changes continuously, the velocity itself is changing. Therefore, the object is continuously accelerating.



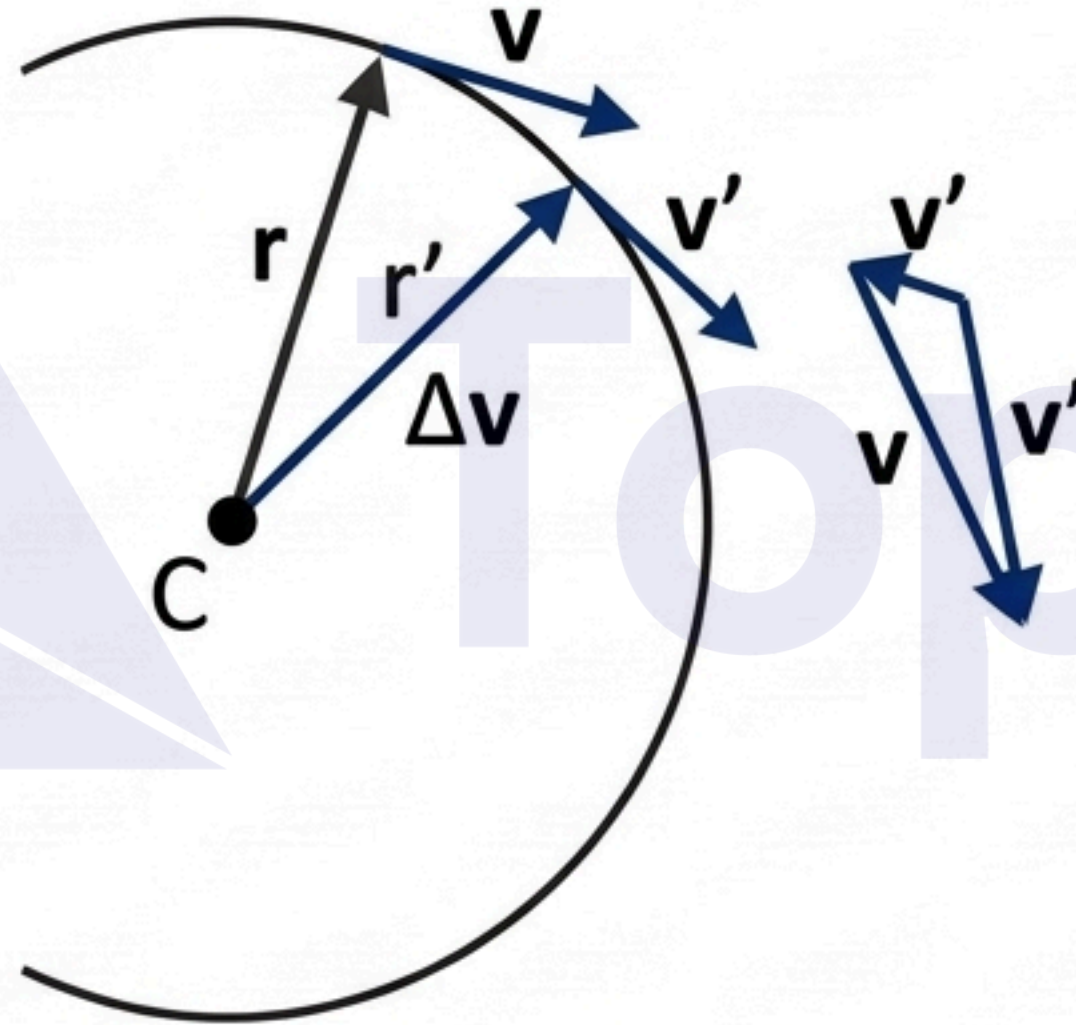
$|v|$ = constant speed ; $\hat{v}$ = changing direction

The Limiting Process: $\Delta t \rightarrow 0$

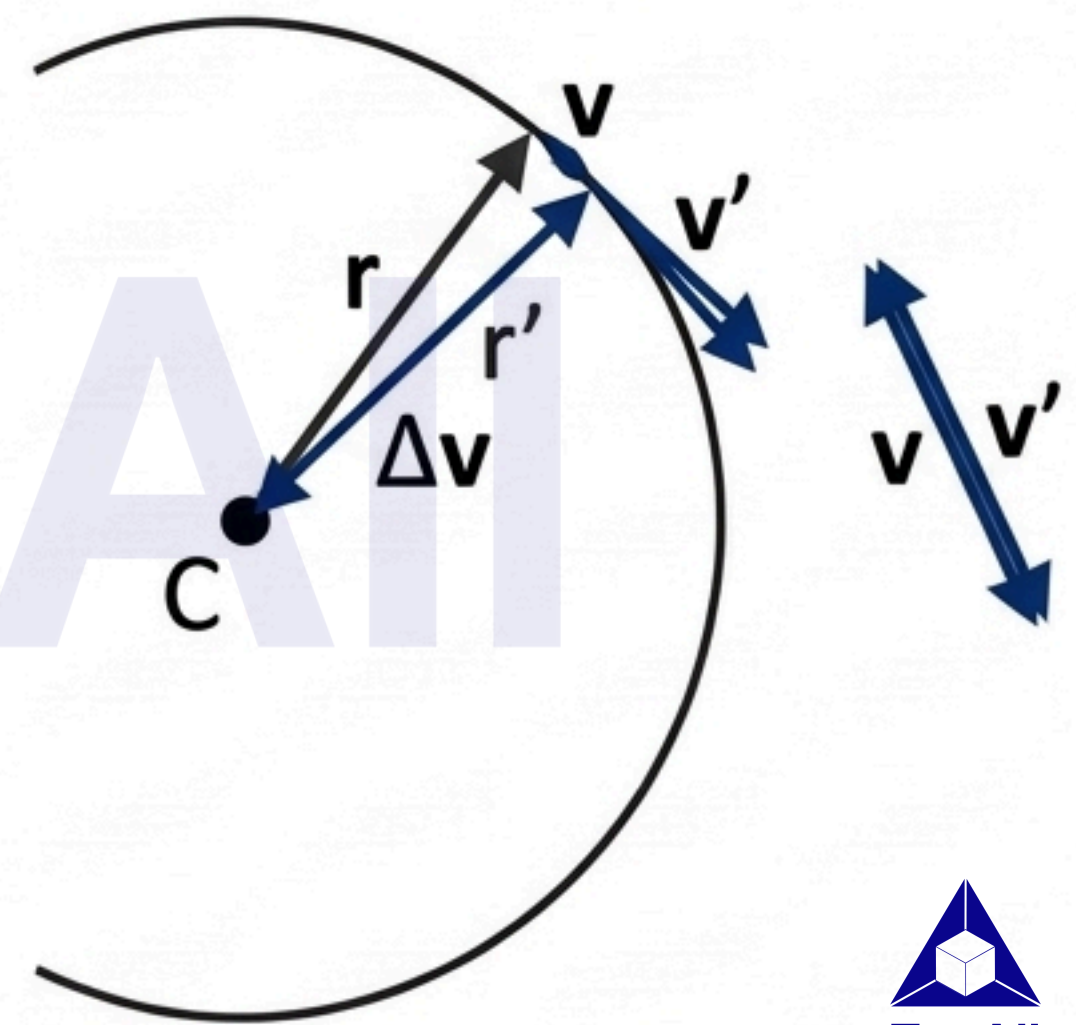
Large Δt



Smaller Δt



$\Delta t \rightarrow 0$



As $\Delta t \rightarrow 0$, the average acceleration becomes the instantaneous acceleration, directed perfectly toward the centre.



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$$E=mc^2$$



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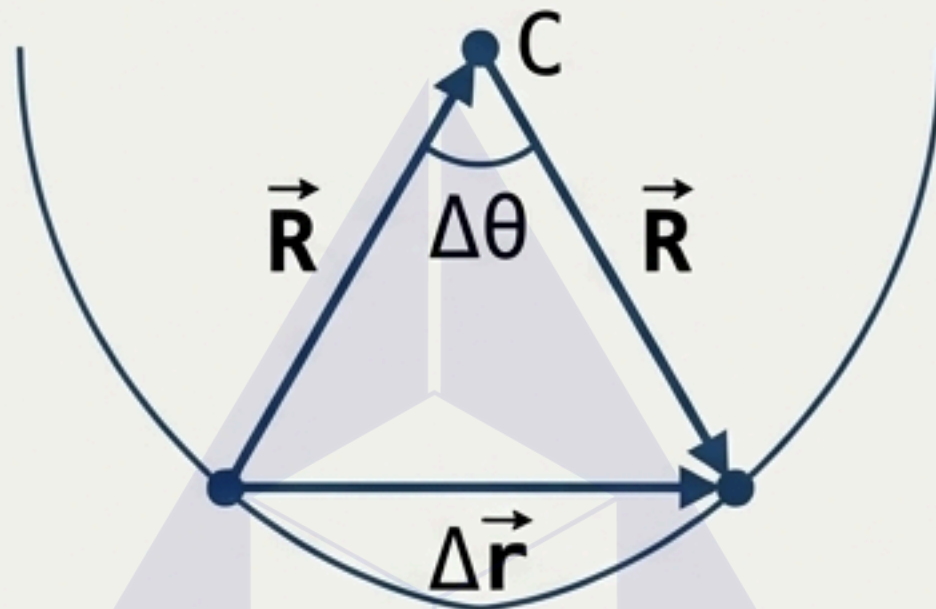
Key Highlights

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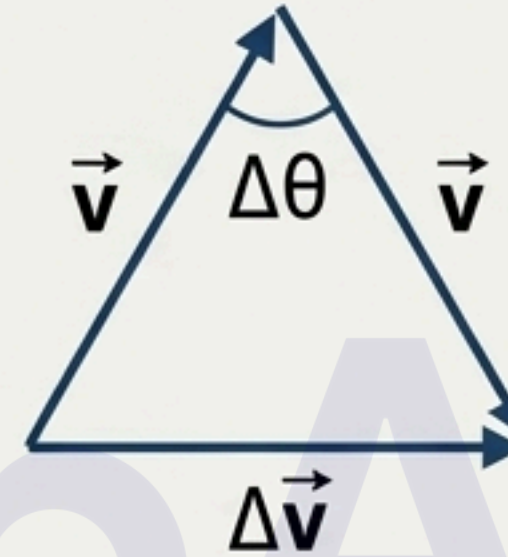


The Geometric Proof (Similar Triangles)

Position Triangle



Velocity Triangle



$$\Delta v / v = \Delta r / R$$

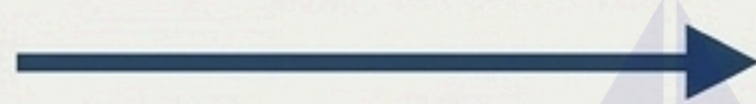
$$\Delta v = v (\Delta r / R)$$

$$a_c = \lim_{(\Delta t \rightarrow 0)} (\Delta v / \Delta t) = (v / R) \lim_{(\Delta t \rightarrow 0)} (\Delta r / \Delta t)$$

$$\mathbf{a_c = v^2 / R}$$

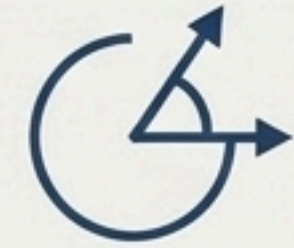
Bridging Linear and Angular Descriptions

Linear



Distance traveled: Δs

Linear Speed:
 $v = \Delta s / \Delta t$



Angular

Angular Distance: $\Delta \theta$

Angular Speed:
 $\omega = \Delta \theta / \Delta t$


$$\Delta s = R \Delta \theta$$

Substituting this yields the master bridge equation:

$$v = R\omega$$

The Core Formula Grid

Linear Acceleration

$$a_c = \frac{v^2}{R}$$

Angular Velocity

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Linear-Angular Bridge

$$v = R\omega$$

Angular Acceleration Bridge

$$a_c = \omega^2 R$$

Frequency-Velocity

$$v = 2\pi R\nu$$

Frequency-Acceleration

$$a_c = 4\pi^2\nu^2 R$$

Where T is time period and $\nu = 1/T$ is frequency.



TRAP ALERT: The 'Constant' Misconception

Centripetal acceleration's MAGNITUDE is constant (v^2/R), but its DIRECTION keeps changing to always point to the centre.

Because direction changes continuously, centripetal acceleration is NOT a constant vector. Therefore, the standard straight-line kinematic equations (like $v = u + at$) DO NOT APPLY to uniform circular motion.

Worked Example 3.9

An insect trapped in a circular groove of radius $R = 12$ cm moves steadily and completes 7 revolutions in 100 s. Find the angular speed (ω), linear speed (v), and the magnitude of acceleration (a_c).

$$\omega = 2\pi / T = 2\pi \times (7 / 100) = 0.44 \text{ rad/s}$$

$$v = \omega R = 0.44 \times 12 = 5.3 \text{ cm/s}$$

$$a_c = \omega^2 R = (0.44)^2 \times 12 \approx 2.3 \text{ cm/s}^2$$

Acceleration magnitude is 2.3 cm/s^2 , pointing to the center.

Key Takeaways: Uniform Circular Motion



- ✓ Centripetal acceleration (a_c) always points to the centre. It has a constant magnitude but an ever-changing direction.
- ✓ The fundamental bridge connecting linear and angular kinematics is $v = R\omega$.
- ✓ Centripetal acceleration can be equivalently expressed as $a_c = v^2/R$ or $a_c = \omega^2 R$.



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