

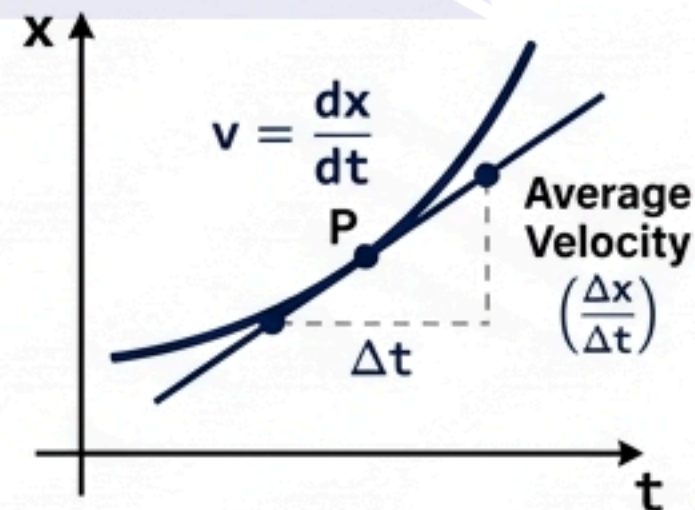
Instantaneous Velocity and Speed

From Average Rates to Calculus-Based Motion
 $\Delta t \rightarrow 0$

INSTANTANEOUS VELOCITY

- Velocity at a specific instant in time.
- Requires calculus for precise definition (derivatives).
- It is a **vector quantity** (has magnitude and direction).

$$\mathbf{v}(t) = \frac{dx}{dt}$$



INSTANTANEOUS SPEED

- The magnitude of instantaneous velocity.
- Always a non-negative scalar quantity.
- Represents how fast an object is moving at an instant, regardless of direction.

$$\text{Speed}(t) = |\mathbf{v}(t)| = \left| \frac{dx}{dt} \right|$$



Key Distinction: **Velocity** is a **vector**, **Speed** is a **scalar**.

The Macroscopic View

Tells us how fast an object moved over a finite time interval, but hides how the **speed varied** during that journey.

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

The Microscopic View

Tells us how fast an object is moving at one exact instant. Defined as the **limit** of average velocity as the time interval becomes infinitesimally small.

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$\lim_{\Delta t \rightarrow 0}$

In the language of calculus, velocity is the differential coefficient of position (x) with respect to time (t).



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$$E=mc^2$$



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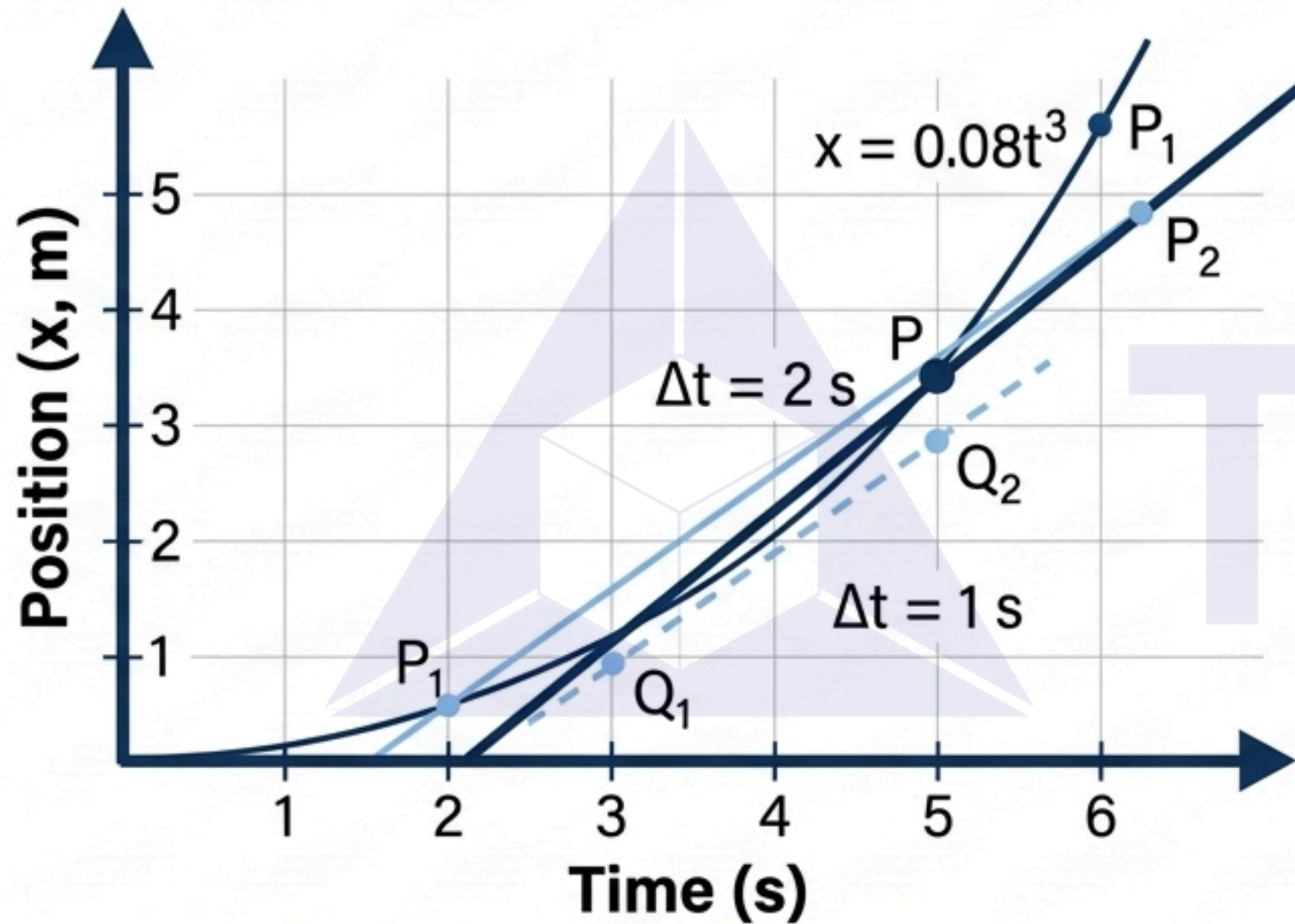
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THE LIMIT OF THE LENS: AVERAGE TO INSTANTANEOUS VELOCITY



As Δt shrinks, the secant line P_1P_2 approaches the tangent at P . Velocity is the slope of this tangent.

Δt (s)	Average Velocity ($\Delta x / \Delta t$, m s ⁻¹)
$\Delta t = 2.0$ s	$\Delta x / \Delta t = 4.16$ m s ⁻¹
$\Delta t = 1.0$ s	$\Delta x / \Delta t = 4.00$ m s ⁻¹
$\Delta t = 0.5$ s	$\Delta x / \Delta t = 3.92$ m s ⁻¹
$\Delta t = 0.1$ s	$\Delta x / \Delta t = 3.85$ m s ⁻¹
$\Delta t = 0.01$ s	$\Delta x / \Delta t = 3.84$ m s ⁻¹

The average velocity approaches exactly 3.84 m s⁻¹ as $\Delta t \rightarrow 0$.

THE LIMIT OF THE LENS: MCQ CARD

The position of an object moving along the x -axis is given by $x = a + bt^2$, where $a = 8.5 \text{ m}$, $b = 2.5 \text{ m s}^{-2}$, and t is in seconds.

- 1. Find instantaneous velocity at $t = 0 \text{ s}$ and $t = 2.0 \text{ s}$.
- 2. Find average velocity between $t = 2.0 \text{ s}$ and $t = 4.0 \text{ s}$.

Step 1: Differentiate for Instantaneous Velocity

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2)$$

$$v = 0 + 2bt = 2(2.5)t = 5.0t \text{ m s}^{-1}$$

Step 2: Evaluate at Instants

$$\text{At } t = 0 \text{ s, } v = 5.0(0) = 0 \text{ m s}^{-1}$$

$$\text{At } t = 2.0 \text{ s, } v = 5.0(2.0) = 10 \text{ m s}^{-1}$$

Step 3: Calculate Average Velocity (Algebraic Interval)

$$x(4.0) = a + b(16) = a + 40$$

$$x(2.0) = a + b(4) = a + 10$$

$$v_{avg} = 30b / 2.0 = 15(2.5) = 15 \text{ m s}^{-1}$$



TRAP ALERT: The Magnitude Paradox

Students often confuse the rules for average rates with the rules for instantaneous rates.

- Macroscopic (Intervals): Average speed is $\geq$ the magnitude of average velocity. (Because path length can be greater than displacement).
- Microscopic (Instants): Instantaneous speed is EXACTLY the magnitude of instantaneous velocity.

There is NO inequality at an exact instant.
A velocity of $+24 \text{ m s}^{-1}$ and -24 m s^{-1} both have an instantaneous speed of exactly 24 m s^{-1} .

EXAM ESSENTIALS: Concept 1



- **The Limit Definition:** Velocity at an instant is the differential coefficient of position with respect to time.

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- **Graphical Meaning:** Instantaneous velocity is the exact slope of the tangent to the position-time (x-t) graph at that specific instant.
- **Speed vs. Velocity:** Instantaneous speed always exactly equals the magnitude of instantaneous velocity. No exceptions.
- **Uniform Motion Shortcut:** For purely uniform motion, instantaneous velocity is identical to average velocity at all instants.



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