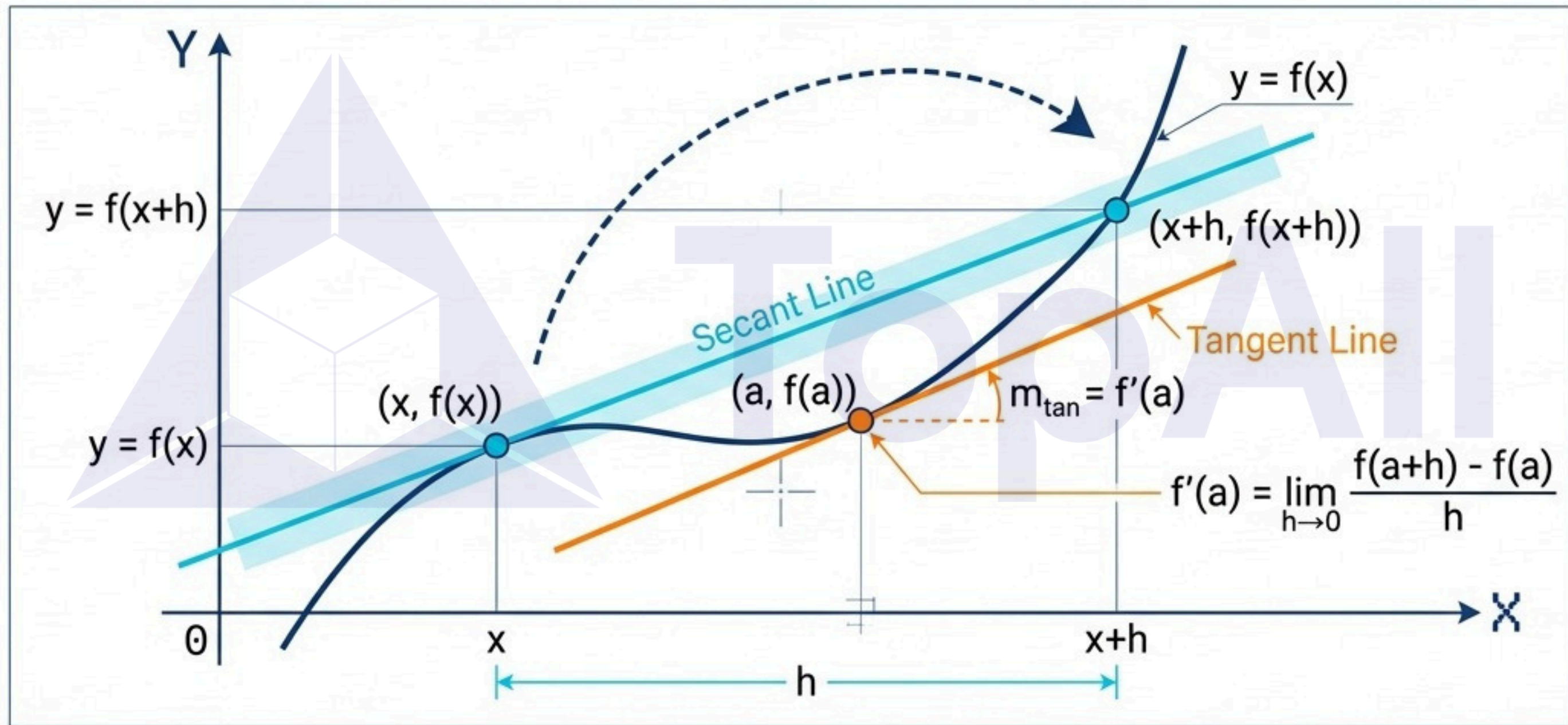


Derivative from First Principle

Calculus Concept 12.5 | Conceptual Foundation & Application



The derivative is the instantaneous rate of change, defined as the limit of the secant line's slope as the interval ' h ' approaches zero, resulting in the tangent line's slope.

Core Concept: Rate of Change at a Point

Average requires an interval. Instantaneous requires a derivative.

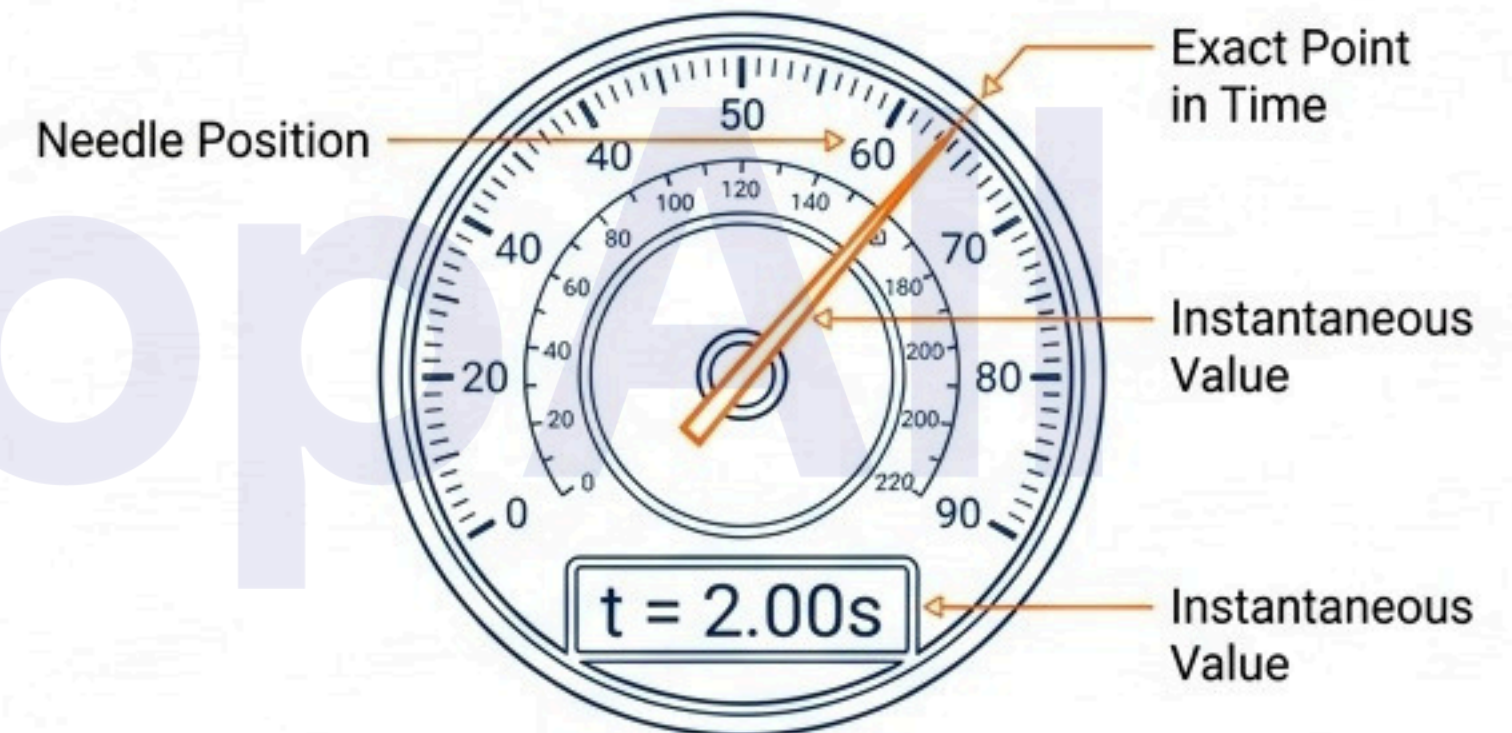
Average Rate (Interval)



Calculates Average Velocity

The Problem: We cannot simply divide distance by zero time to find the exact speed at a frozen instant.

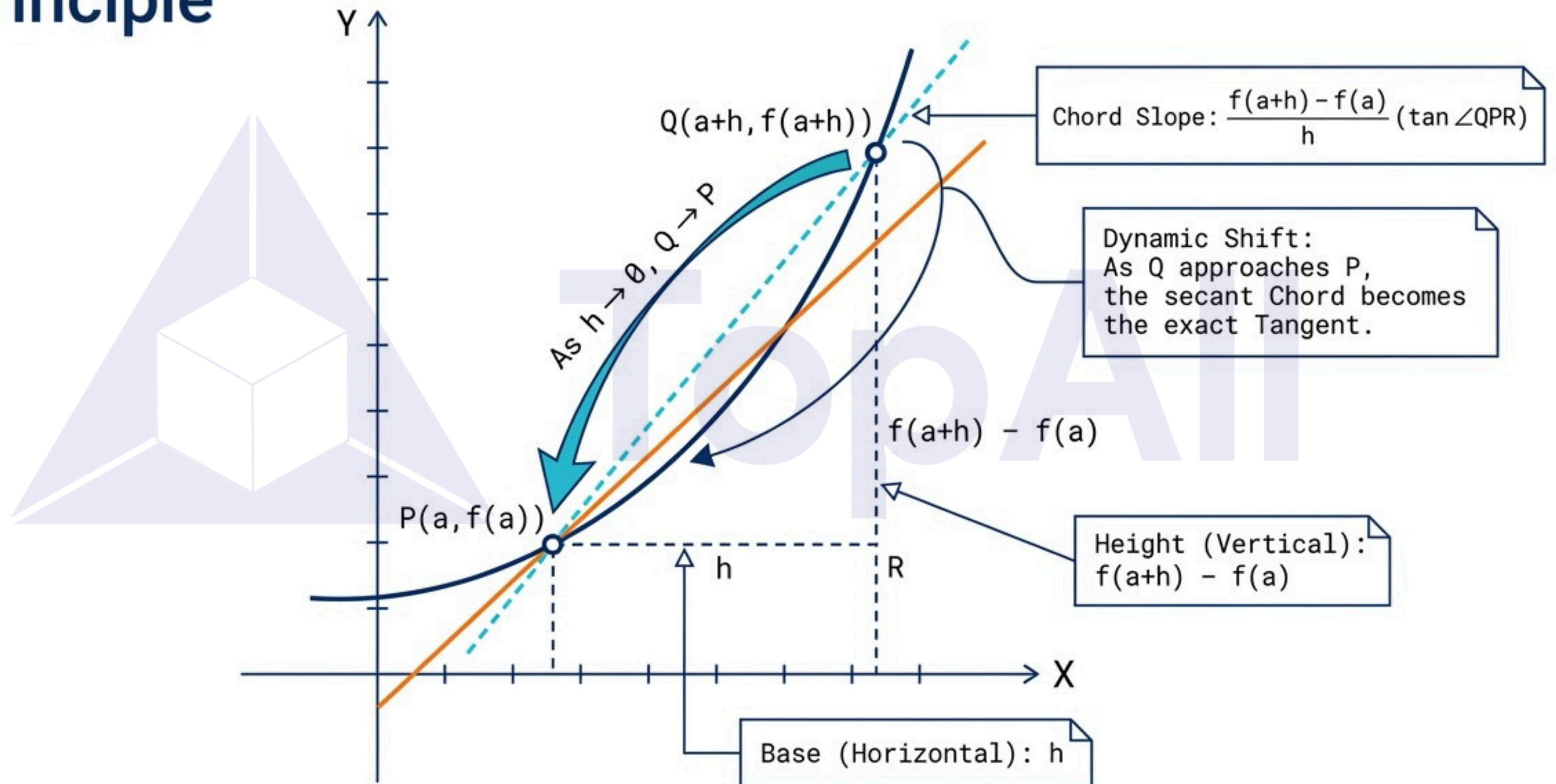
Instantaneous Rate (Point)



Calculates Instantaneous Velocity

The Solution: We calculate the average rate over a time interval h , then shrink h until it is infinitesimally close to 0. The limit of this process is the Derivative.

The Geometry of the First Principle





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Formula Bank: First Principle of Derivatives

The Derivative: The exact slope of the **tangent line** at $x = a$.

Change in Y: The vertical difference (Height).

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

The Limit Operator: Forces the two distant points on the curve to merge into one single point.

Change in X: The shrinking horizontal interval (**Base**).

Execution Flaws & Pitfalls

⚠ JEE TRAP ALERT: Do not substitute $h = 0$ before algebraic simplification!

The Mistake (Indeterminate Form)

$$\frac{f(a + \textcolor{red}{0}) - f(a)}{\textcolor{red}{0}}$$

$$\rightarrow \frac{f(a) - f(a)}{\textcolor{red}{0}}$$

$$\rightarrow \boxed{\frac{0}{0}}$$

Indeterminate Form - Dead End!

The Fix (Execution Protocol)

1. Expand the numerator term $f(a + h)$.

2. Cancel out the original $f(a)$ terms.

3. Factor out h from the remaining numerator.

4. Eliminate the h in the denominator.

5. Limit: Only then apply $h \rightarrow 0$.

Mini Application: Deriving $f(x) = x^2$

Goal: Prove $(x^2)' = 2x$ using First Principles.

Step 1: Set up the limit formula

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

Step 2: Expand the numerator

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\cancel{x^2} + 2xh + \cancel{h^2}) - \cancel{x^2}}{h}$$

Step 3: Simplify (Cancel)

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

Step 4: Factor and eliminate h

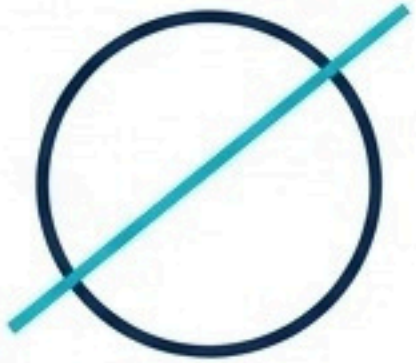
$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + \cancel{h})}{\cancel{h}} = \lim_{h \rightarrow 0} (2x + h)$$

Step 5: Apply the limit

$$f'(x) = 2x + 0 = \mathbf{2x}$$

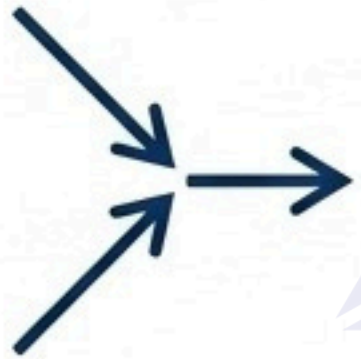
Synthesis & Summary

KEY TAKEAWAYS



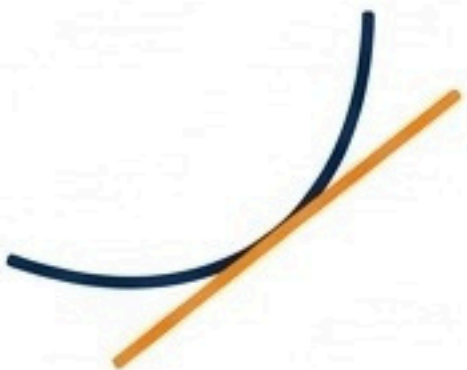
1. The Geometric Translation

The Difference Quotient $\left[\frac{f(a+h) - f(a)}{h} \right]$ represents the slope of the secant chord.



2. The Limiting Process

Applying the limit operator $[h \rightarrow 0]$ shrinks the horizontal interval to zero, physically transforming the secant chord into the tangent line.



3. The Algebraic Result

The output of this limiting process is the Derivative $[f'(a)]$, which provides the exact, instantaneous rate of change at point a .



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