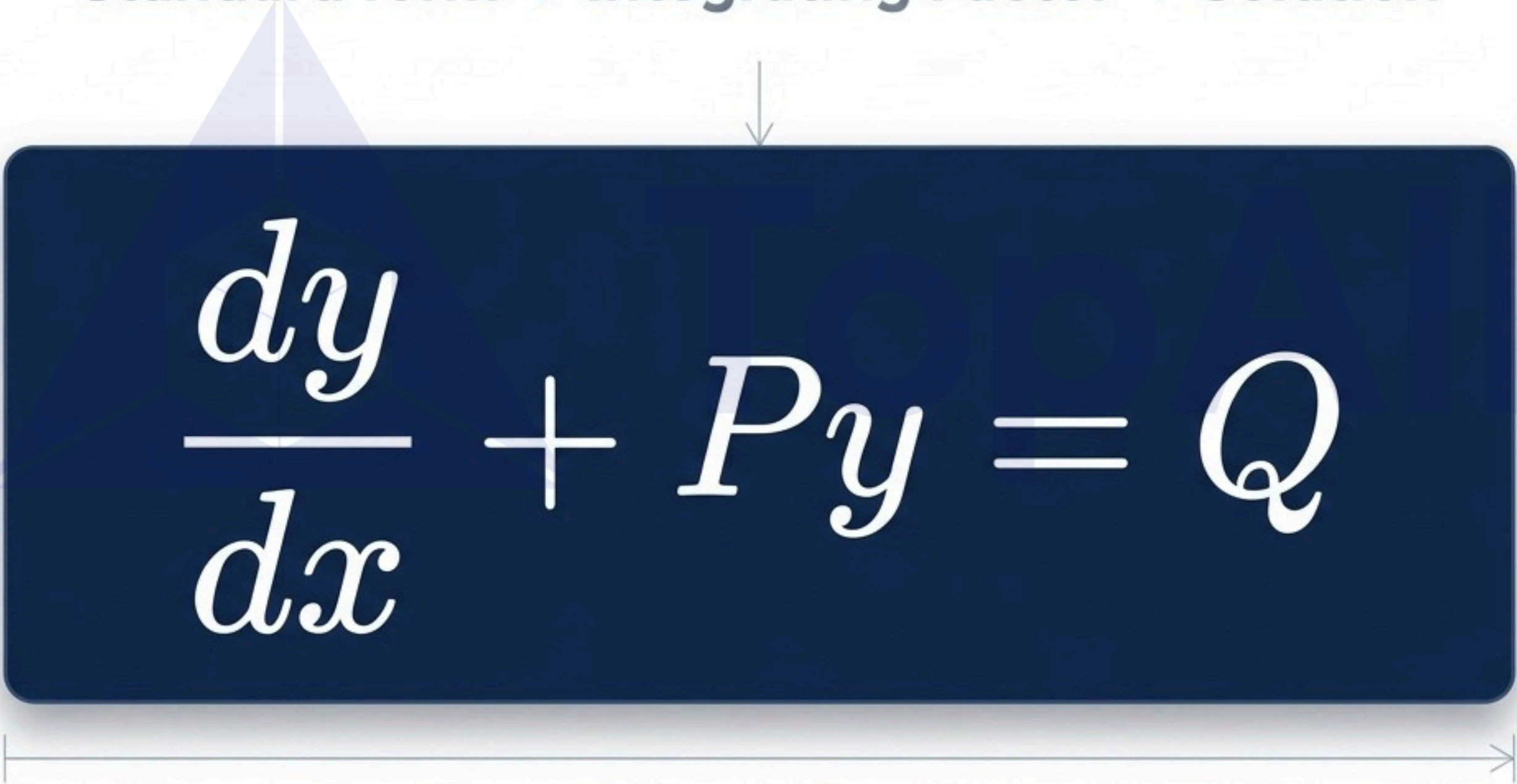
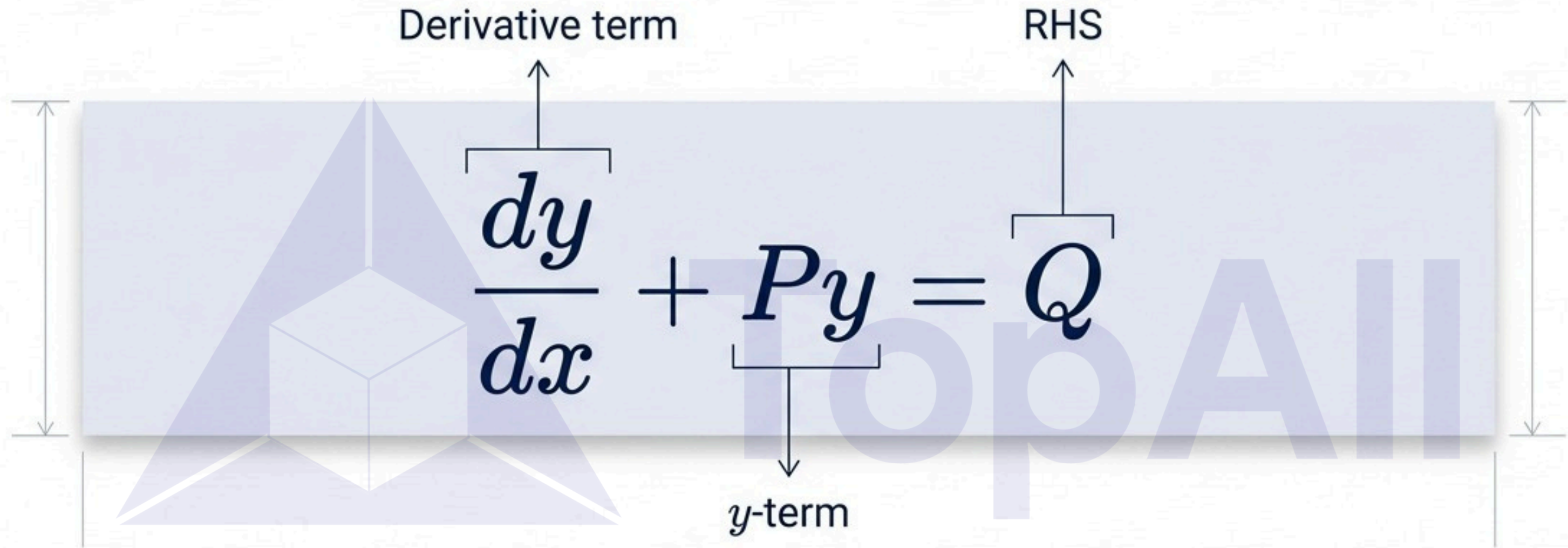


Linear Differential Equations

Standard form → Integrating Factor → Solution


$$\frac{dy}{dx} + Py = Q$$

Recognise the Linear Form



The diagram shows the linear differential equation $\frac{dy}{dx} + Py = Q$ enclosed in a light blue rectangular box. Above the box, the text "Derivative term" has an arrow pointing to the $\frac{dy}{dx}$ term, which is bracketed. To the right, the text "RHS" has an arrow pointing to the Q term, which is also bracketed. Below the box, the text " y -term" has an arrow pointing to the Py term, which is bracketed. Vertical double-headed arrows are on the left and right sides of the box. A large, faint "TopAll" watermark is visible in the background.

$$\frac{dy}{dx} + Py = Q$$

Derivative term

RHS

y -term

where P and Q are constants or functions of x only.

$$\frac{dx}{dy} + P_1x = Q_1 \quad \text{where } P_1 \text{ and } Q_1 \text{ depend on } y \text{ only.}$$

Make the Left Side One Derivative

$$\frac{dy}{dx} + Py = Q$$

Multiply by $g(x)$

$$g(x)\frac{dy}{dx} + P \cdot g(x)y = Q \cdot g(x)$$

Goal: Force the left side to perfectly match the Product Rule expansion:

$$\frac{d}{dx}[y \cdot g(x)] = g(x)\frac{dy}{dx} + y \cdot g'(x)$$



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Where Does the I.F. Formula Come From?

Compare terms from the previous slide:

$$P \cdot g(x)y = y \cdot g'(x)$$

$$\Rightarrow P \cdot g(x) = g'(x)$$

$$\Rightarrow P = \frac{g'(x)}{g(x)}$$

$$\int P dx = \log(g(x))$$

INTEGRATING FACTOR

$$\text{I.F.} = e^{\int P dx}$$

The Standard Solution Formula

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + C$$



Make the Derivative Coefficient 1 First

$$x \frac{dy}{dx} + 2y = x^2 \quad (x \neq 0)$$

$$P = 2$$



Incorrect: Derivative coefficient is not 1.

Divide by x



$$\frac{dy}{dx} + \frac{2}{x}y = x$$

$$P = \frac{2}{x}, \quad Q = x$$



Example: Find the Integrating Factor

From: $\frac{dy}{dx} + \frac{2}{x}y = x$

Identify: $P = \frac{2}{x}$

$$\begin{aligned}\text{I.F.} &= e^{\int \frac{2}{x} dx} \\ &= e^{2 \log x} \\ &= x^2\end{aligned}$$

Substitute into standard formula:

$$y(x^2) = \int x \cdot x^2 dx + C$$

$$yx^2 = x^4/4 + C$$

Linear Differential Equation in 5 Steps

1. Write in standard form: $\frac{dy}{dx} + Py = Q$
2. Identify P and Q
3. Find I.F.: $I.F. = e^{\int P dx}$
4. Multiply/apply: $y(I.F.)$
5. Integrate: $y(I.F.) = \int Q(I.F.) dx + C$

Alternative Form:

$$\frac{dy}{dy} + P_1 x = Q_1$$

uses $I.F. = e^{\int P_1 dy}$

Memory Line
Normalise first. Integrating factor second.



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