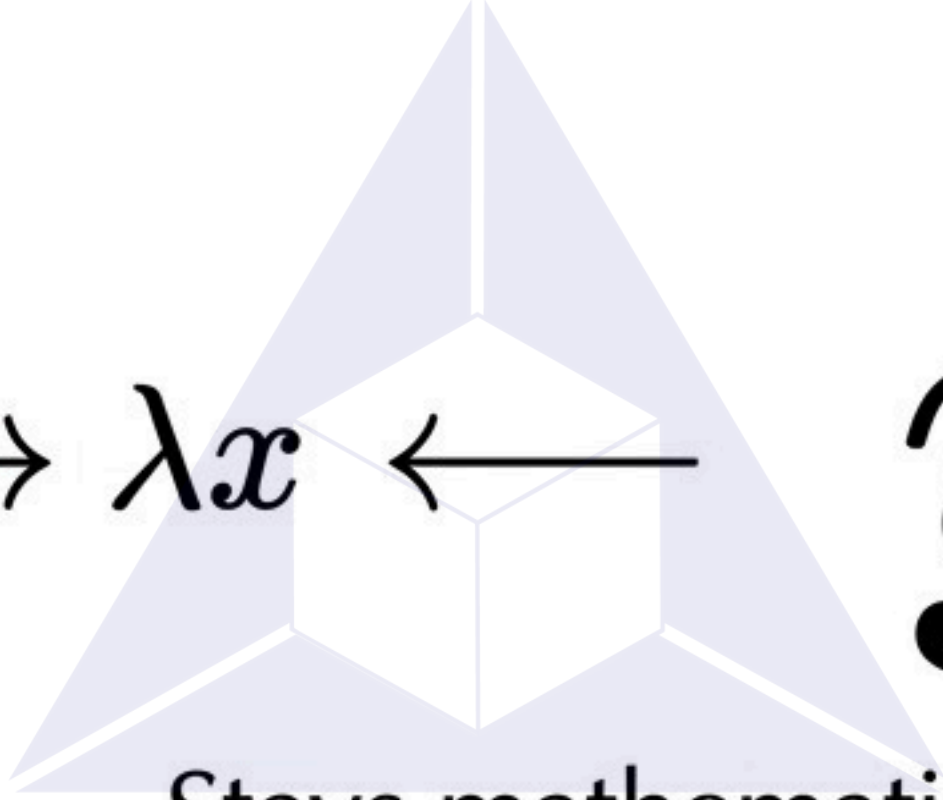


Homogeneous Differential Equations

Recognise the ratio. Make the substitution.


$$x \rightarrow \lambda x \quad \leftarrow \quad y/x \quad \longrightarrow \quad y \rightarrow \lambda y$$

Stays mathematically unchanged when scaled.

$$\frac{\cancel{\lambda y}}{\cancel{\lambda x}} = \frac{y}{x}$$

First Understand the Function

A function is homogeneous of degree n if scaling its inputs scales its output by λ^n .

$$F(\lambda x, \lambda y) = \lambda^n F(x, y)$$

Homogeneous function of degree n

Function	Scaling Test	Verdict
$F(x, y) = y^2 + 2xy$	$(\lambda y)^2 + 2(\lambda x)(\lambda y) = \lambda^2(y^2 + 2xy)$	Degree 2
$F(x, y) = 2x - 3y$	$2(\lambda x) - 3(\lambda y) = \lambda(2x - 3y)$	Degree 1
$F(x, y) = \cos(y/x)$	$\cos(\lambda y/\lambda x) = \lambda^0 \cos(y/x)$	Degree 0
$F(x, y) = \sin x + \cos y$	$\sin(\lambda x) + \cos(\lambda y) \neq \lambda^n F(x, y)$	Non-homogeneous

When Is the Differential Equation Homogeneous?

The derivative must equal a homogeneous function of degree zero.

$$\frac{dy}{dx} = F(x, y)$$

If $F(x, y)$ is homogeneous of degree zero (λ^0),
then the equation is homogeneous.

Equivalent Recognition Pattern

By factoring out x^n or y^n , a degree zero function can always be rearranged entirely in terms of ratios.

$$\frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

$x \neq 0$



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Replace the Ratio

Transform the complex (x, y) relationship into a solvable (v, x) relationship.

Step 1

$$\frac{y}{x} = v \Rightarrow y = vx$$

Step 2

Differentiate with respect to x (Product Rule)

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$
$$\frac{dy}{dx} = v \cdot (1) + x \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$



Do NOT write $\frac{dy}{dx} = v$.

Because y is a product of two variables (v and x), you must apply the product rule.

The Substitution Unlocks the Equation

Watch how the x and v variables neatly separate.

Start (Original Form): $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$

Substitute (Apply Slide 4 logic): $v + x \frac{dv}{dx} = g(v)$

Isolate Derivative: $x \frac{dv}{dx} = g(v) - v$

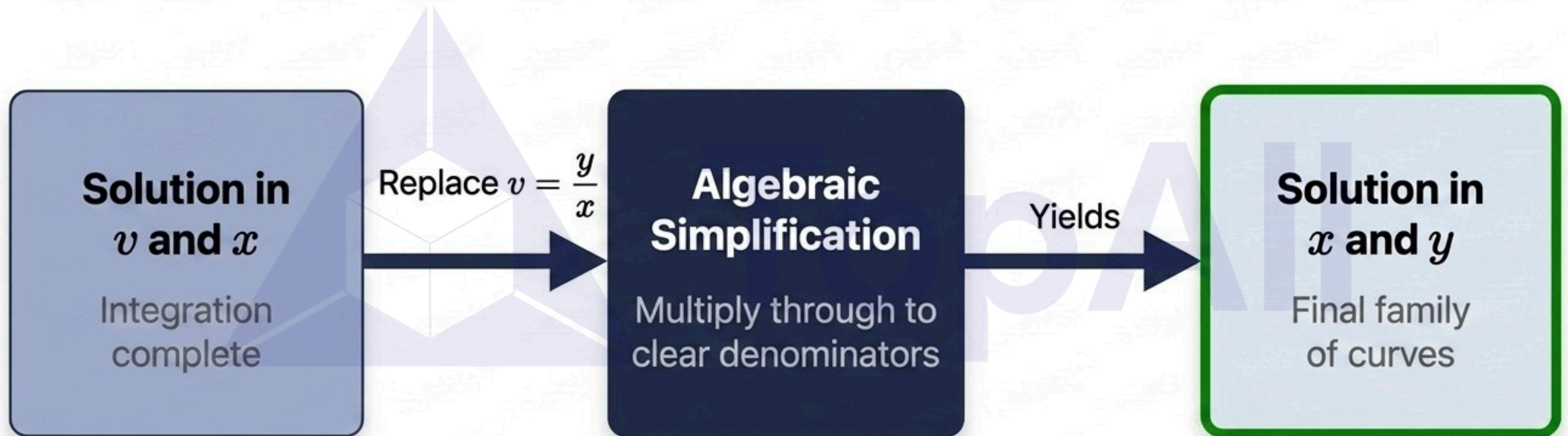
Separate Variables: $\frac{dv}{g(v) - v} = \frac{dx}{x}$

Integrate:

$$\int \frac{dv}{g(v) - v} = \int \frac{dx}{x} + C$$

Never Leave the Answer in v

The problem was asked in x and y . The solution must be in x and y .



Example: Recognise y/x

Applying the substitution to a standard differential equation.

Problem: $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

Divide numerator and denominator by x^2 : $\frac{dy}{dx} = \frac{1 + (y/x)^2}{2(y/x)}$

Let $y = vx$. Then $\frac{dy}{dx} = v + x \frac{dv}{dx}$

$$v + x \frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{1 + v^2}{2v} - v = \frac{1 + v^2 - 2v^2}{2v} = \frac{1 - v^2}{2v}$$

Separate & Integrate

$$\int \frac{2v}{1 - v^2} dv = \int \frac{dx}{x}$$

$$-\log|1 - v^2| = \log|x| + \log|C_1|$$

Substitute Back $v = y/x$ and simplify

$$x^2 - y^2 = Cx$$

Homogeneous DE in One Frame

Standard Pattern

Recognise: $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$

Use substitution: $y = vx$

Remember
Product Rule: $\frac{dy}{dx} = v + x \frac{dv}{dx}$

Workflow: Separate $\rightarrow$ Integrate $\rightarrow$
Replace $v = \frac{y}{x}$

Alternative Form

Recognise: If the equation naturally
forms $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$
(e.g., when y is isolated in an exponent like $e^{x/y}$)

Use substitution: $x = vy$

Remember
Product Rule: $\frac{dx}{dy} = v + y \frac{dv}{dy}$

Workflow: Separate $\rightarrow$ Integrate $\rightarrow$
Replace $v = \frac{x}{y}$



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