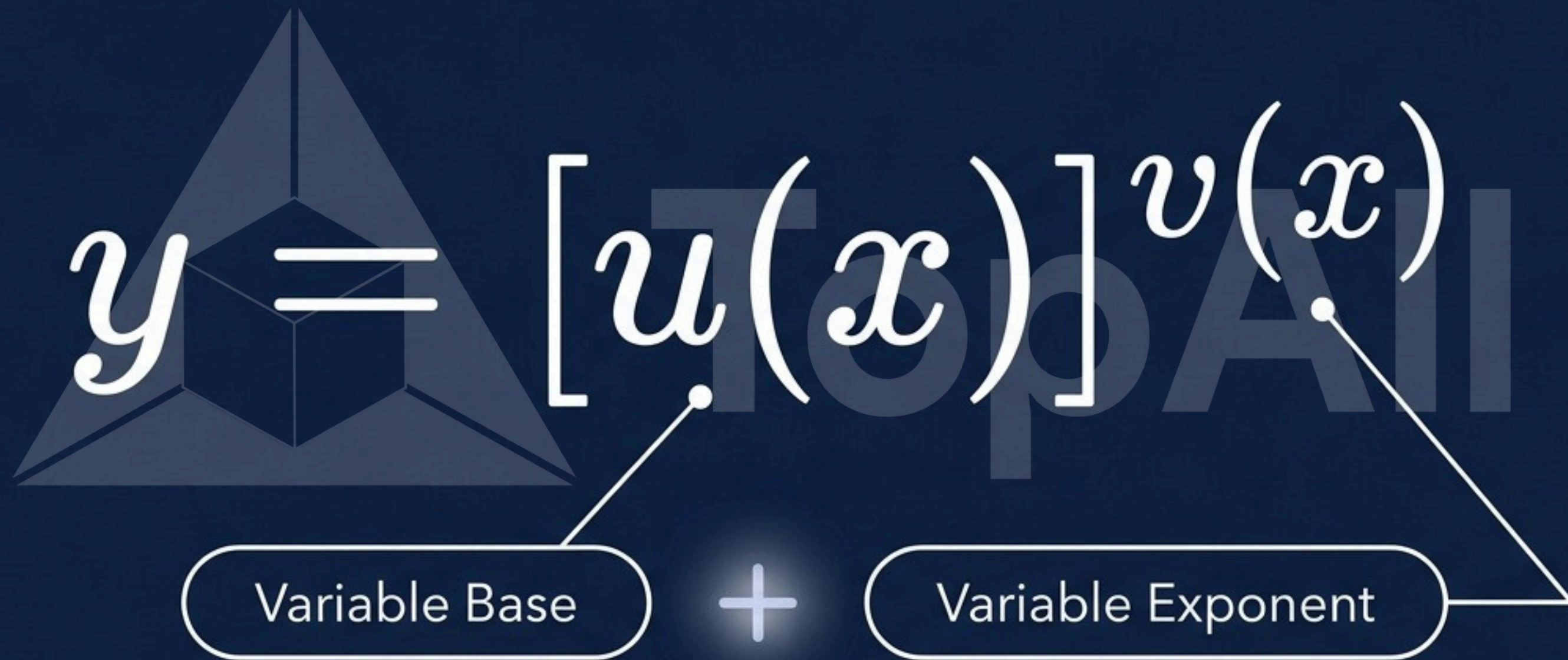


Logarithmic Differentiation

The method for functions with variable powers



The diagram illustrates the logarithmic differentiation formula $y = [u(x)]^{v(x)}$. A large, faint 3D pyramid is visible in the background. The equation is written in white. Below the equation, two labels in rounded rectangles are connected to the equation by lines. The label 'Variable Base' is connected to the $u(x)$ term, and the label 'Variable Exponent' is connected to the $v(x)$ term. A plus sign is placed between the two labels.

$$y = [u(x)]^{v(x)}$$

Variable Base + Variable Exponent

What Makes This Form Different?

$$x^5$$

Variable base,
constant exponent.

Solved by the
Power Rule.

$$5^x$$

Constant base,
variable exponent.

Solved by
Exponential Rules.

$$x^{\sin x}$$

Both base and
exponent change.

Standard rules fail.

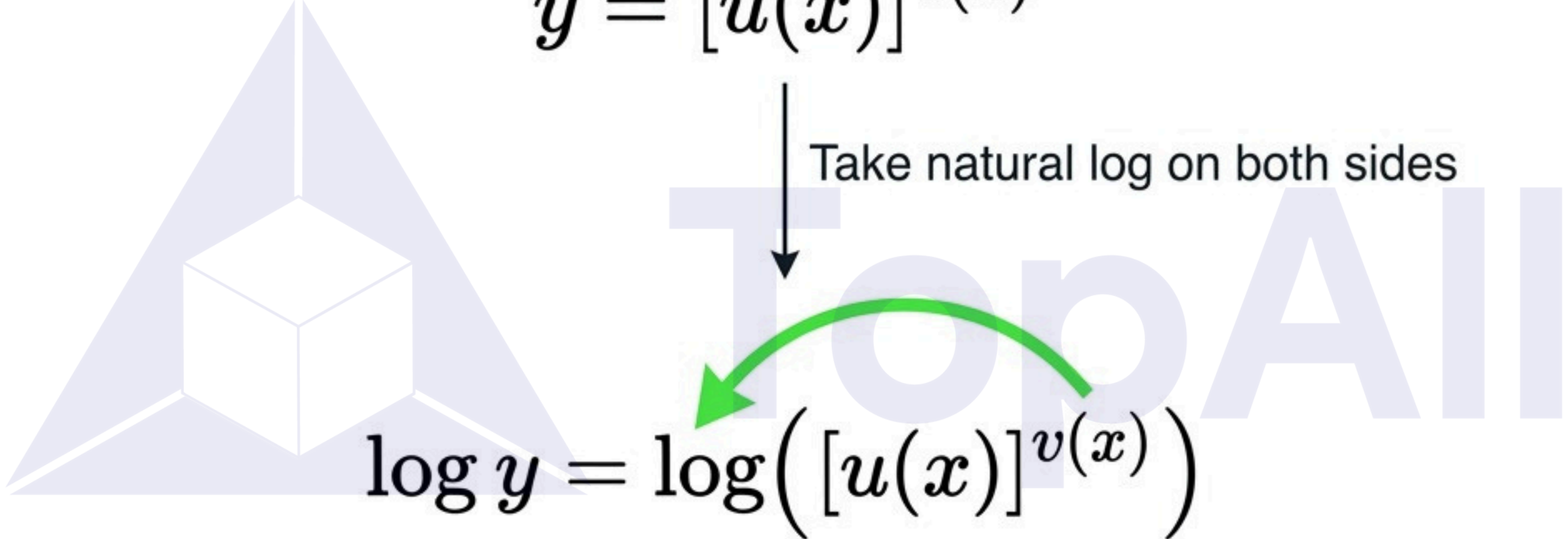
Take Logarithms First

The logarithm's power property converts a difficult exponent into a simple product.



$$y = [u(x)]^{v(x)}$$

Take natural log on both sides

A diagram illustrating the step of taking the natural logarithm of both sides. A vertical black arrow points from the equation $y = [u(x)]^{v(x)}$ down to the equation $\log y = \log([u(x)]^{v(x)})$. A green curved arrow points from the text "Take natural log on both sides" down to the same equation. A large, faint "TopAll" watermark is visible in the background.
$$\log y = \log([u(x)]^{v(x)})$$

$$\log y = v(x) \cdot \log[u(x)]$$



**You've Learned the Theory.
Let's Test Them Now.**

$$E=mc^2$$



Turn theory into marks with **unlimited MCQs, smart analysis, and focused revision.**

Practice Now



At prep.topall.app

Key Highlights

- » Free Study Material, Videos & Infographics
- » Push Your Limits with Level-Based Challenges
- » Climb the Leaderboard Among Fellow Aspirants
- » Practice Unlimited MCQs for NEET & JEE
- » Get Clear Explanations, Not Just Marks
- » Track Your Performance Smarter



Now Ordinary Rules Take Over

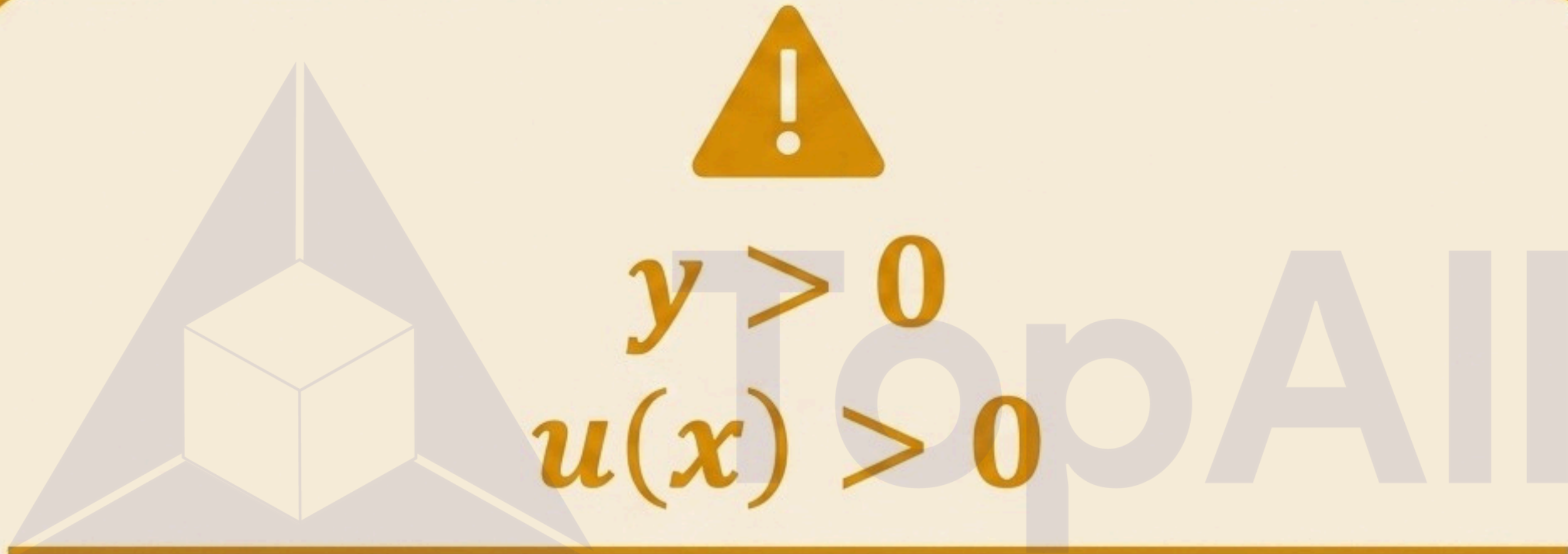
Differentiate both sides with respect to x .

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{v(x)}{u(x)} \cdot u'(x) + v'(x) \cdot \log[u(x)]$$

Product Rule applied

$$\frac{dy}{dx} = y \left[v(x) \frac{u'(x)}{u(x)} + v'(x) \log u(x) \right]$$

A Condition You Cannot Ignore



For this textbook method to be valid, the base function and the overall expression must be strictly positive.

Logarithms are not defined for zero or negative numbers.

Differentiate $x^{\sin x}$

Given

$$y = x^{\sin x} \text{ (where } x > 0\text{)}$$

Take log

$$\log y = \sin x \cdot \log x$$

Differentiate

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \cos x \cdot \log x$$

Therefore

$$\frac{dy}{dx} = x^{\sin x} \left[\frac{\sin x}{x} + \cos x \log x \right]$$

The 5-Step Method

1

Set $y =$ expression

2

Take log on both sides

3

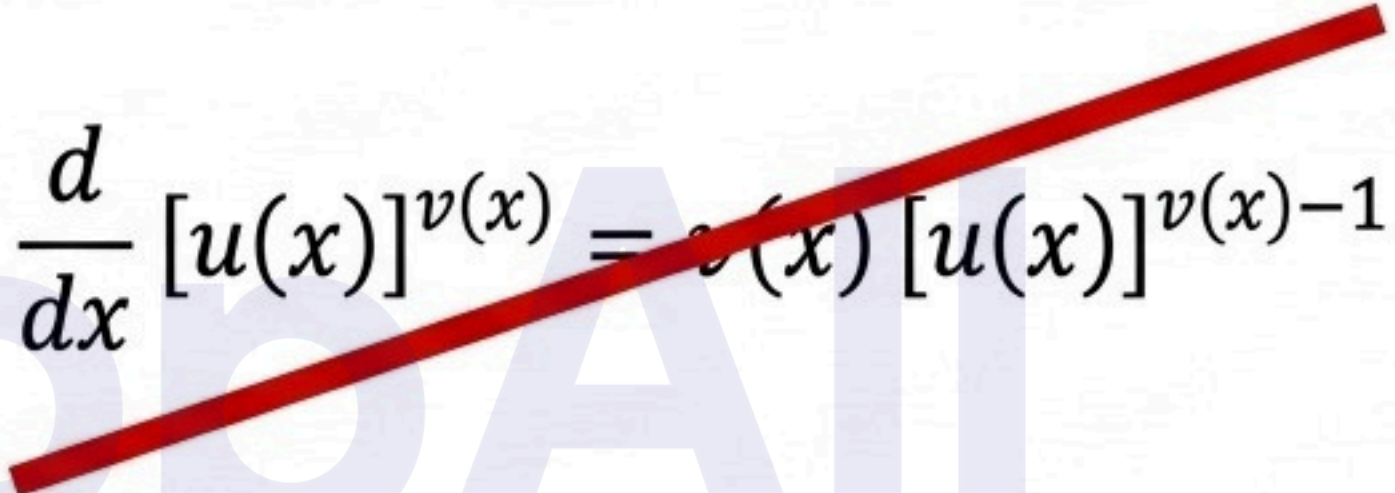
Use log rules to move exponent

4

Differentiate implicitly

5

Replace y with original expression


$$\frac{d}{dx} [u(x)]^{v(x)} = v(x) [u(x)]^{v(x)-1}$$

TRAP: Never apply the power rule to a variable exponent.

VARIABLE POWER? THINK LOGARITHMIC DIFFERENTIATION.



You've Learned the Theory.

Now Let's Test It.

Reading notes gets you knowledge.

Practicing MCQs gets you marks.

That's exactly what TopAll Pro is built for.

Try TopAll Pro Now

What TopAll Pro Does Differently

-  **Stuck? Snap it. Solve it. — Doubts AI**
Photo → step-by-step solution in seconds, 24/7
-  **Your AI Coach finds your blind spots**
Concept-level weakness detection — not just chapter-level
-  **4.5 Lakh+ questions. Never run out.**
13× more content than any other prep app in India
-  **Build your own test in 60 seconds**
Pick a chapter, set difficulty — AI does the rest instantly.
-  **AI Recommendation that helps you improve**
AI Recommendation Analysis your Tests and tell you where you need to improve.



Practice the moment you finish studying
Jump in while the concept is still fresh in your head

4.5L+
MCQs

1.5L+
Students

AI Powered
In-Depth Analysis