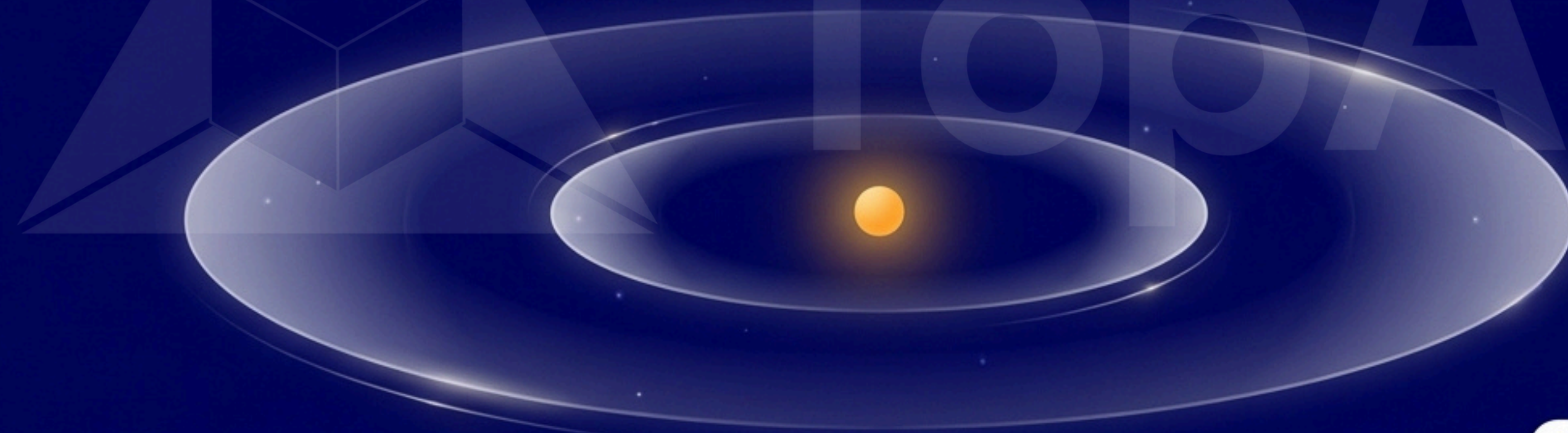


Why Does Point-Charge Potential Fall As $1/r$?

We know the Electric Field falls off steeply as $1/r^2$. But Electrostatic Potential tells a different story. It stretches further, fading slower.

Today, we derive the exact formula for the potential of a point charge from infinity to point P, and discover exactly why this happens.



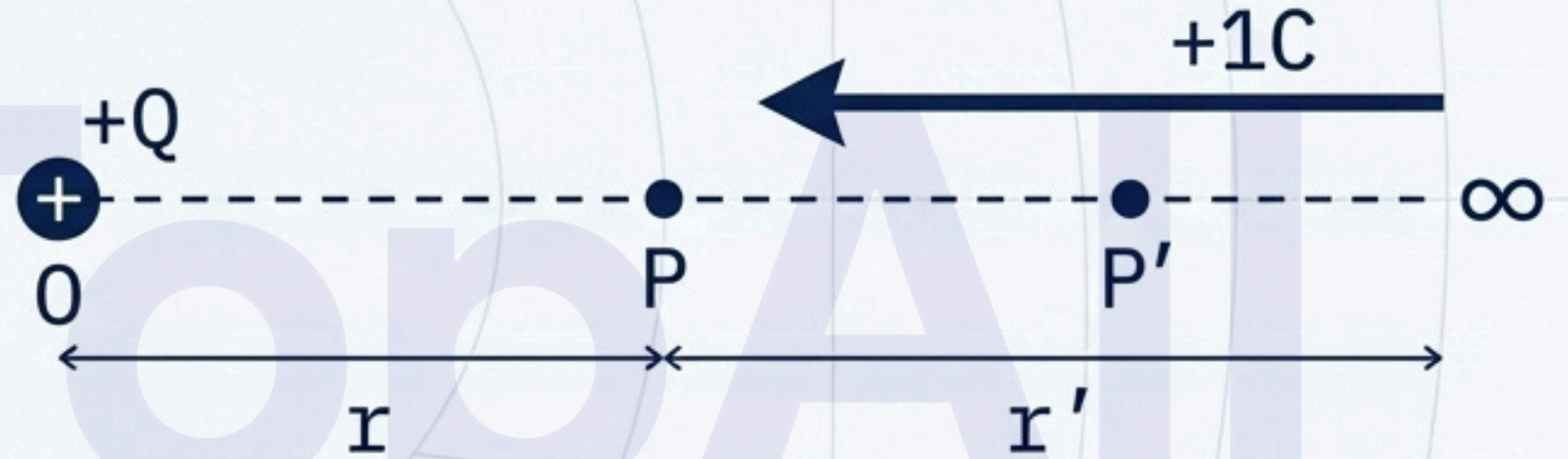
Practice more, score more

The Setup: A Test Charge's Journey From Infinity

Let's place a positive point charge Q at the origin. We want to find the potential at point P (position vector $\mathbf{r}$).

By definition, potential is the work done to bring a unit positive test charge ($+1\text{C}$) from infinity (∞) to point P .

Instructor Note: Imagine the test charge starting infinitely far away, where it feels zero push. As it moves closer, Q repels it. We must do work to push it to point P .



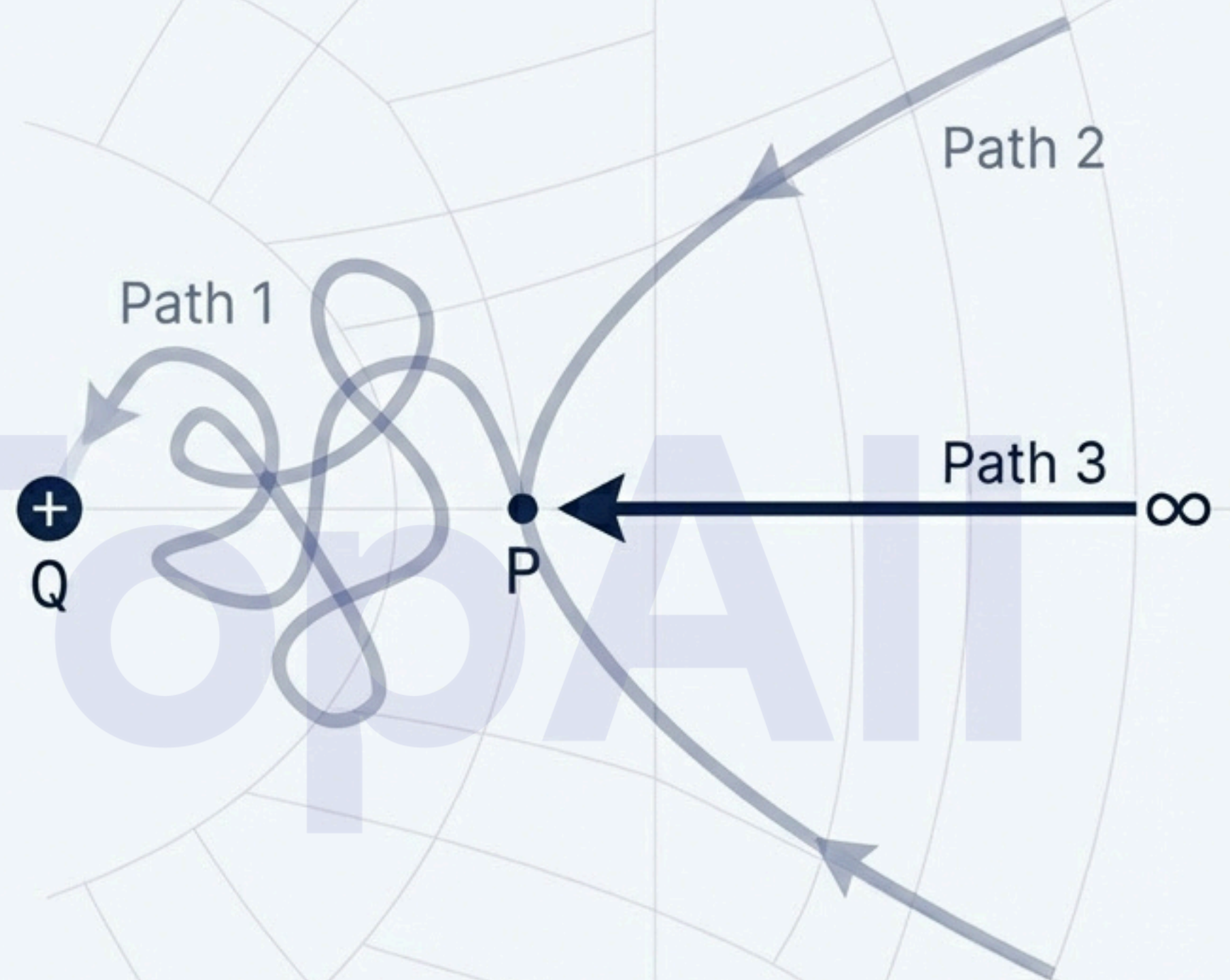
Practice more, score more

The Strategy: Why We Choose The Radial Path

Insight: Electrostatic force is a conservative force.

This means the work done is independent of the path taken. Whether the charge spirals, zigzags, or goes straight—the total work is exactly the same.

To make the math easy, we choose the simplest path: a straight line along the radial direction from infinity to P.



Practice more, score more



**You've Learned the Theory.
Let's Test Them Now.**

$$E=mc^2$$



Turn theory into marks with **unlimited MCQs, smart analysis, and focused revision.**

Practice Now



At prep.topall.app

Key Highlights

- » Free Study Material, Videos & Infographics
- » Push Your Limits with Level-Based Challenges
- » Climb the Leaderboard Among Fellow Aspirants
- » Practice Unlimited MCQs for NEET & JEE
- » Get Clear Explanations, Not Just Marks
- » Track Your Performance Smarter



The Derivation: Step-by-Step

Step 1: Force

At an intermediate point P' at distance r' , the force on $+1C$ is:

$$\mathbf{F} = \frac{Q}{4\pi\epsilon_0 r'^2} \hat{\mathbf{r}}'$$

Step 2: Work Element

Work done against this force for a tiny inward step $\Delta r'$:

$$\Delta W = -\frac{Q}{4\pi\epsilon_0 r'^2} \Delta r'$$

Negative sign because $\Delta r' < 0$

Step 3: Integration

Integrate the total work from $r' = \infty$ to $r' = r$:

$$\begin{aligned} W &= -\int_{\infty}^r \frac{Q}{4\pi\epsilon_0 r'^2} dr' \\ &= \frac{Q}{4\pi\epsilon_0 r} \end{aligned}$$

Instructor Note: First, find force at a random point. Second, find work for a tiny step. Third, integrate to add up all tiny steps from infinity.

Practice more, score more

Potential Due To A Point Charge

$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

This is Equation 2.8. It represents the potential at point P due to charge Q. This is mathematically equal to the total work done by an external force to bring a unit positive charge from infinity to distance r.

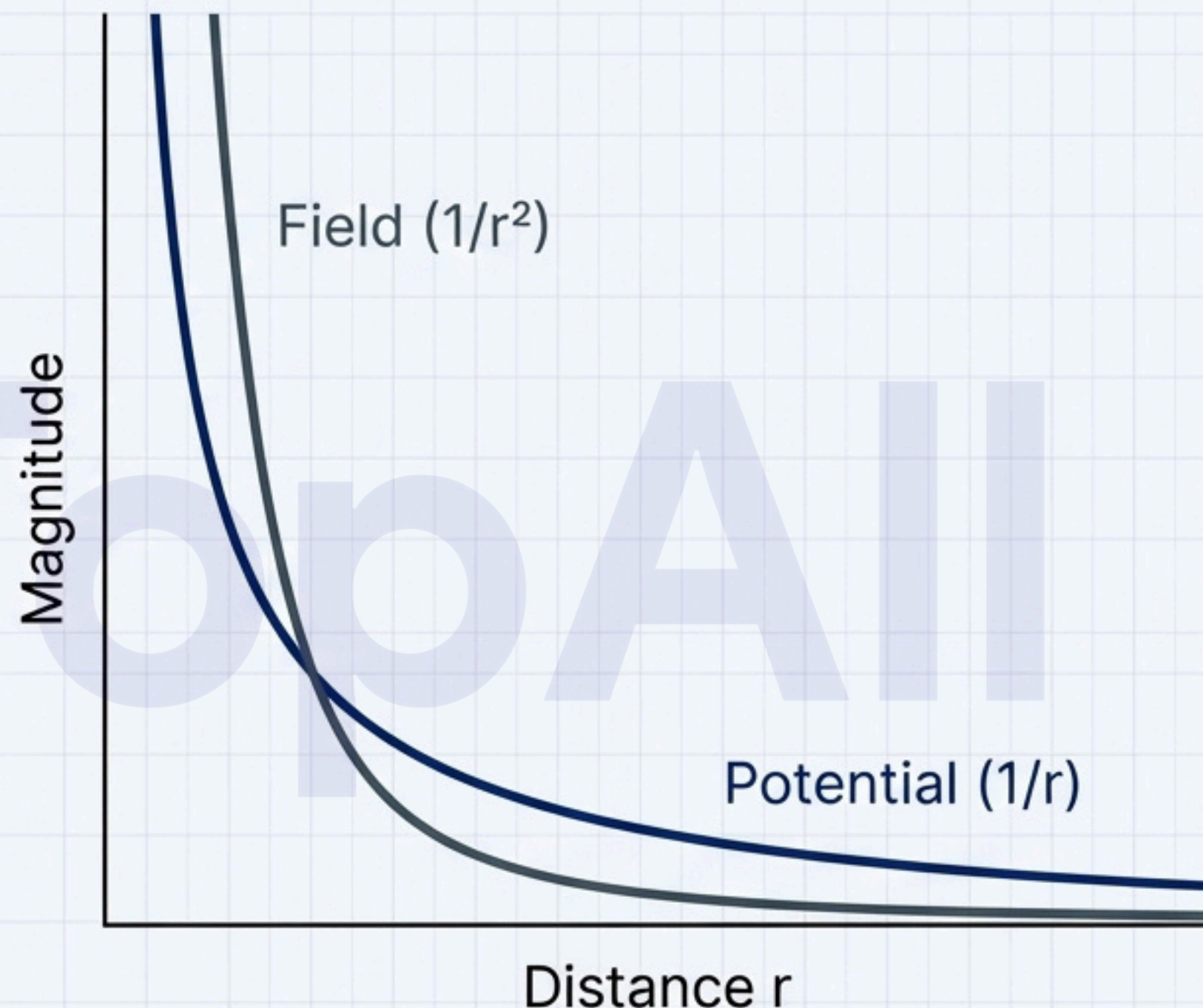
Common Mistake: Do not square the r in the denominator.

Visualizing The Drop: Potential vs. Field

Potential
 $V \propto 1/r$
(Fades slower)

Electric Field
 $E \propto 1/r^2$
(Crashes faster)

While Electric Field drops off sharply, the Potential extends much further out into space.



Key Takeaways & Exam Traps

Watch Your Signs!

- If $Q > 0$, then $V > 0$ (Positive work is done against repulsion).
- If $Q < 0$, then $V < 0$ (Force is attractive; work done by external force per unit charge is negative).

Exam Takeaways

- Remember the exact formula:
 $V = Q / 4\pi\epsilon_0 r$
- Choose infinity (∞) as the zero-potential reference point.
- Distinguish carefully between $V \propto 1/r$ and $E \propto 1/r^2$ on graphical questions.



You've Learned the Theory.

Now Let's Test It.

Reading notes gets you knowledge.

Practicing MCQs gets you marks.

That's exactly what TopAll Pro is built for.

Try TopAll Pro Now

What TopAll Pro Does Differently

-  **Stuck? Snap it. Solve it. — Doubts AI**
Photo → step-by-step solution in seconds, 24/7
-  **Your AI Coach finds your blind spots**
Concept-level weakness detection — not just chapter-level
-  **4.5 Lakh+ questions. Never run out.**
13× more content than any other prep app in India
-  **Build your own test in 60 seconds**
Pick a chapter, set difficulty — AI does the rest instantly.
-  **AI Recommendation that helps you improve**
AI Recommendation Analysis your Tests and tell you where you need to improve.



Practice the moment you finish studying
Jump in while the concept is still fresh in your head

4.5L+
MCQs

1.5L+
Students

AI Powered
In-Depth Analysis