

Class 11 Physics • Chapter 6: Systems of Particles

Angular Momentum: Fixed-Axis Rotation

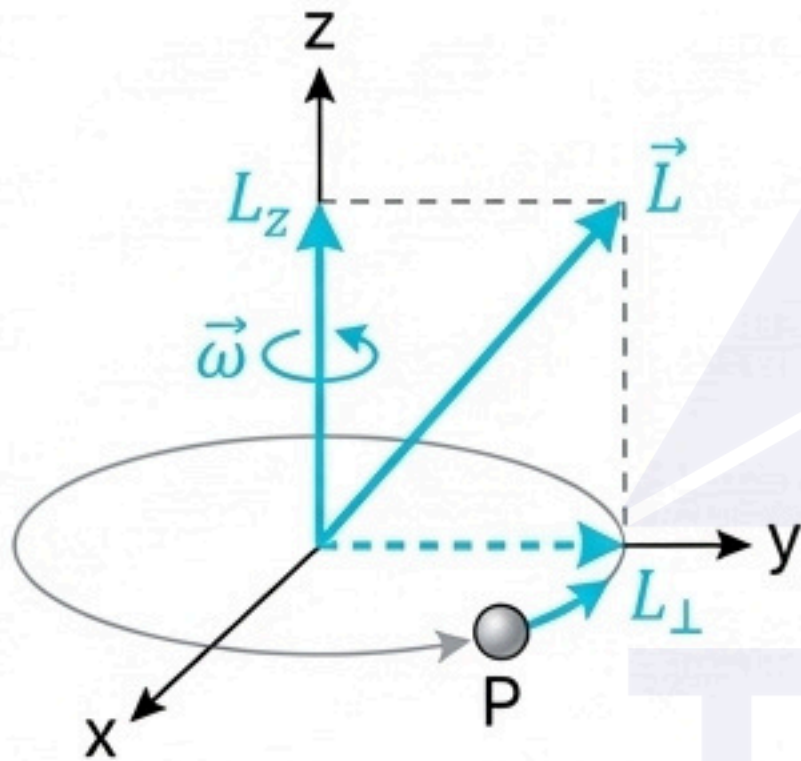
Mastering the dynamics of symmetric rigid bodies and conservation laws.



Taming the Vectors: Why Fixed-Axis?

For a general rotating body, $\vec{L}$ is not always parallel to $\vec{\omega}$. Fixing the axis simplifies the math drastically.

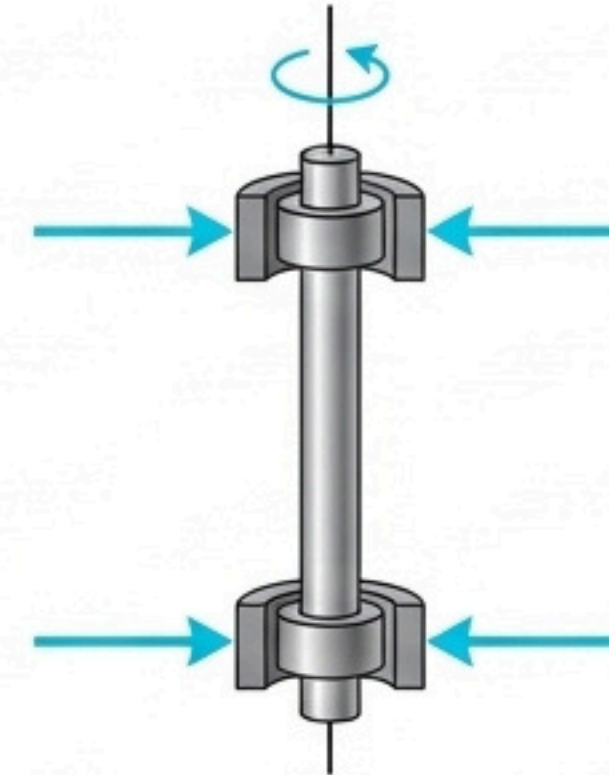
The Complex Reality



$L_\perp$ (**Perpendicular**): Tries to tilt the axis. $\sum L_\perp = 0$.

L_z (**Axial**): The useful component that drives rotation. $L_z = I\omega$.

The 1D Simplification

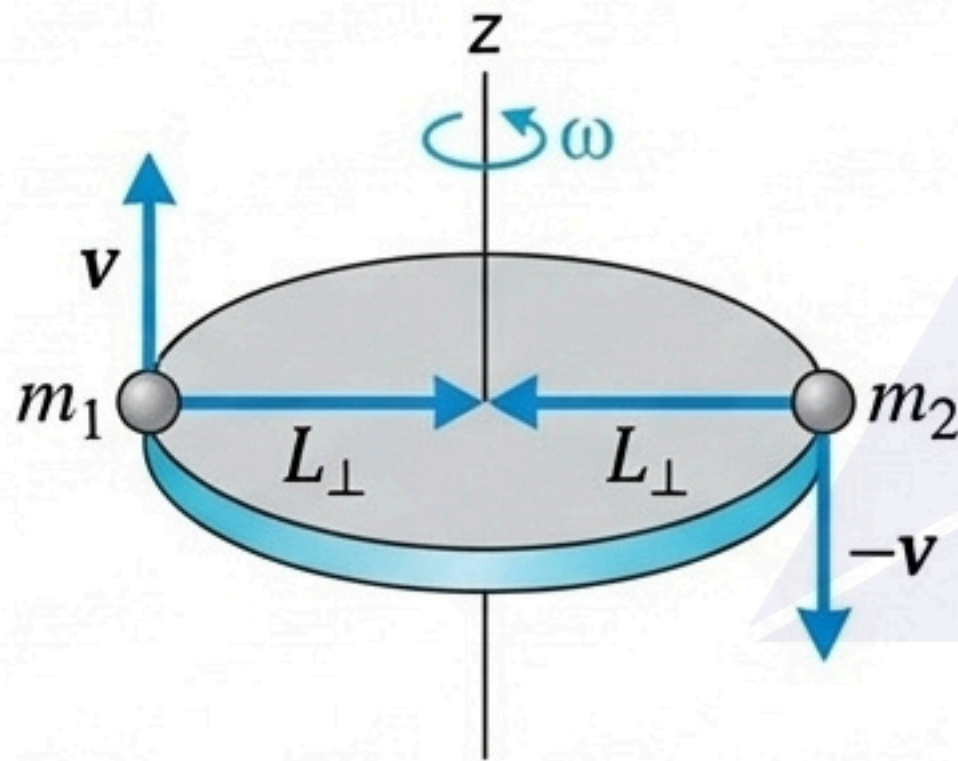


Forces of constraint (hinges/bearings) physically cancel the perpendicular components.

Exam Note: We only care about the components along the axis of rotation!

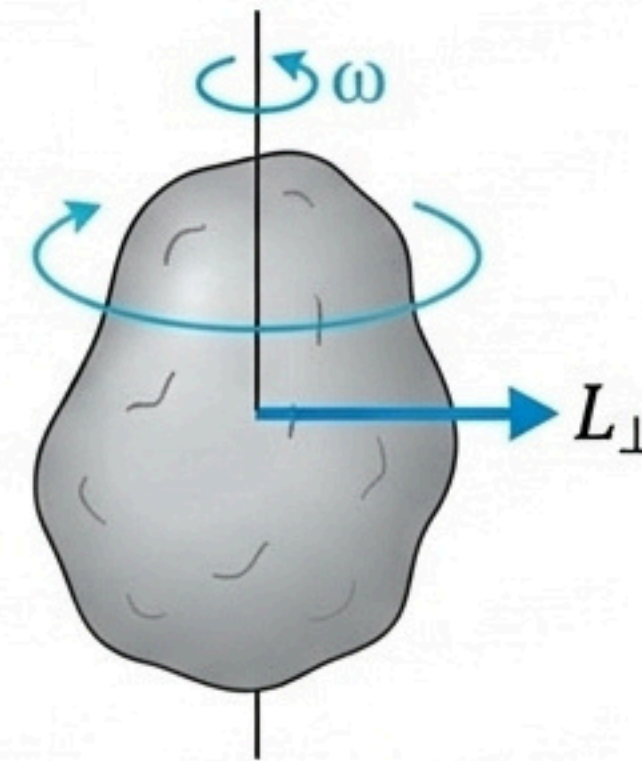
The Power of Symmetry

Symmetric Bodies



For every particle with velocity $\mathbf{v}$, an opposite particle has velocity $-\mathbf{v}$. Result: Perpendicular components completely vanish ($\sum L_{\perp} = 0$).

Asymmetric Bodies



$\mathbf{L}$ and $\boldsymbol{\omega}$ are NOT necessarily parallel. The body wobbles relentlessly unless artificially constrained.

The Magic Equation: $\mathbf{L} = I\boldsymbol{\omega}$

From Kinematics to Dynamics

$$\frac{dL_z}{dt} = \tau$$

Time rate of change of angular momentum equals Torque (Newton's 2nd Law for rotation).

Substitute $L_z = I\omega \Rightarrow \frac{d(I\omega)}{dt} = \tau$

$$I \frac{d\omega}{dt} = \tau$$

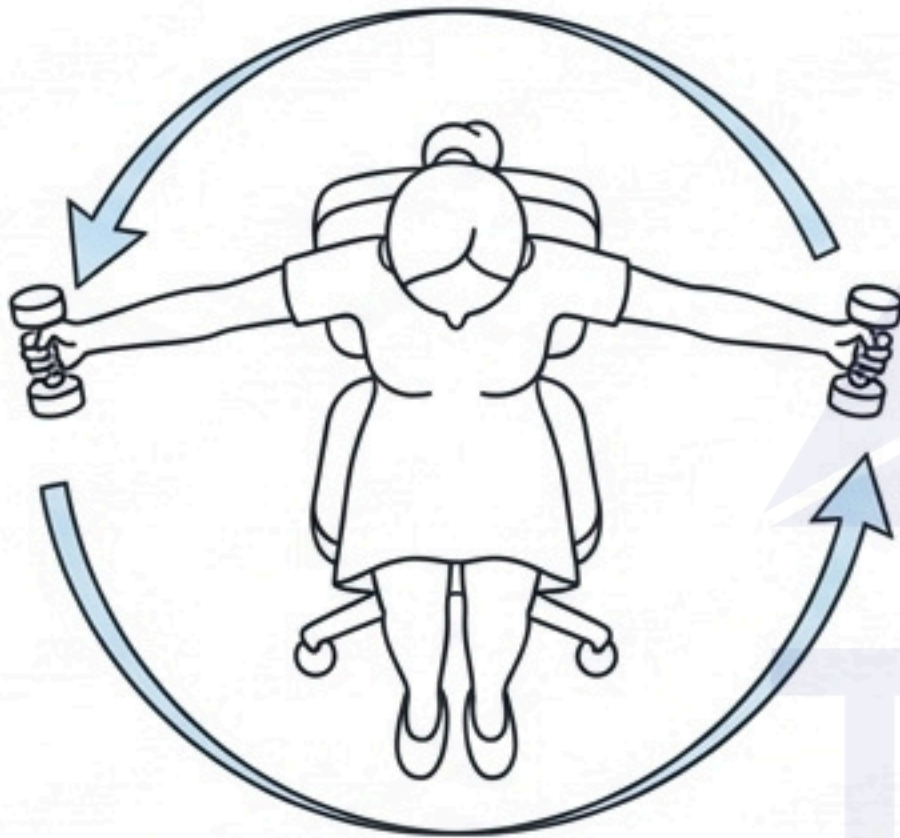
If Moment of Inertia (I) is constant, we can pull it out of the derivative.

Since $\frac{d\omega}{dt} = \alpha$ (angular acceleration), we arrive at the fundamental law of rotational dynamics: $\tau = I\alpha$

Conservation of Angular Momentum

If external torque (τ_{ext}) = 0, then $\vec{L}$ is conserved. Therefore, $I\omega = \text{constant}$.

Arms Out



Moment of Inertia (I) increases
→ Angular speed (ω) decreases.

Arms In



Moment of Inertia (I) decreases
→ Angular speed (ω) increases.

Core Exam Constraint:

$$\tau_{\text{ext}} = 0 \Rightarrow I_1\omega_1 = I_2\omega_2$$