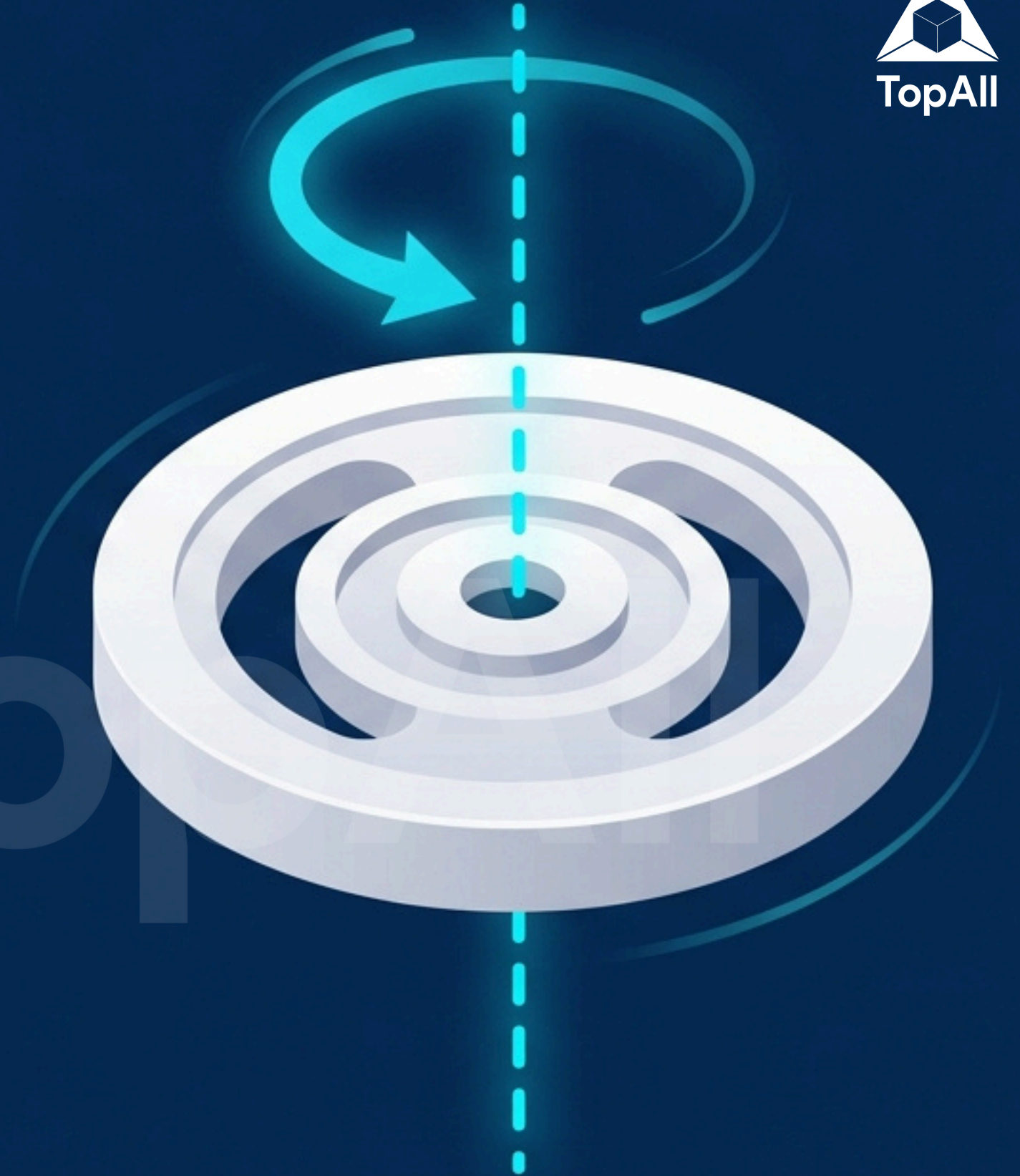


Moment of Inertia

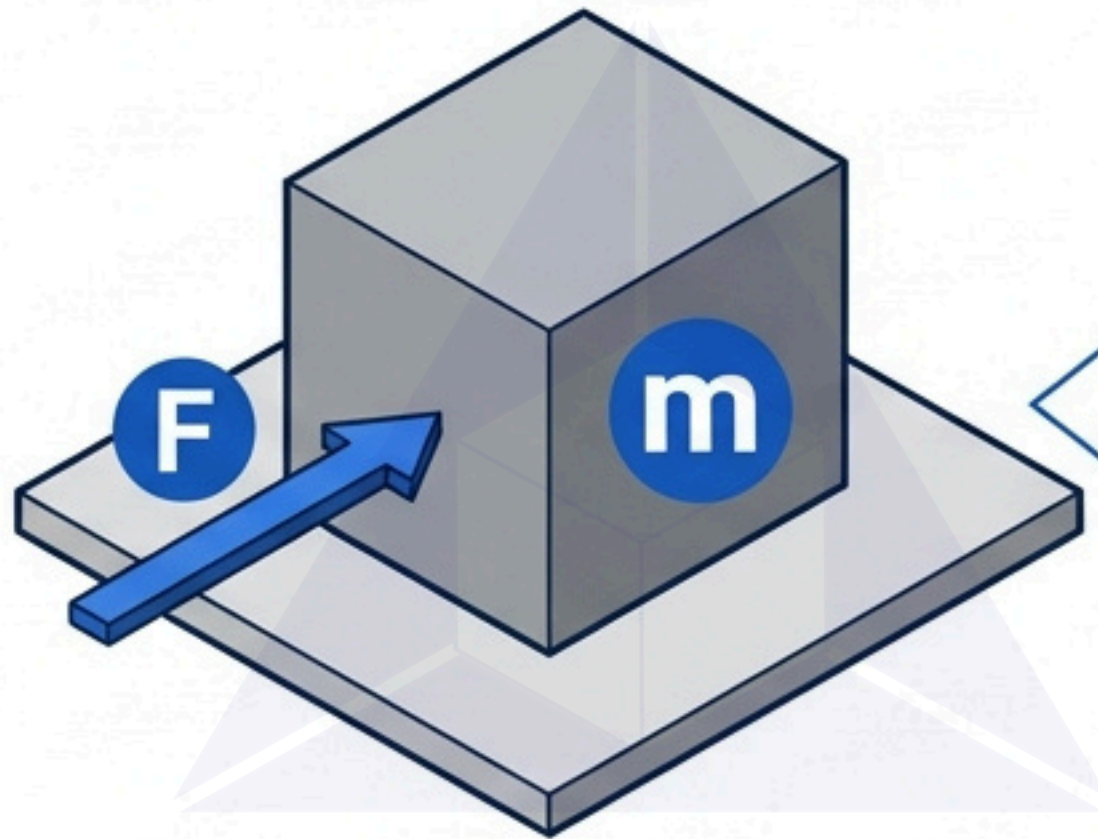
The Architecture of Rotational Resistance



Why Some Bodies Resist Rotation More

In linear motion, mass (m) resists acceleration. In rotational motion, Moment of Inertia (I) resists angular acceleration.

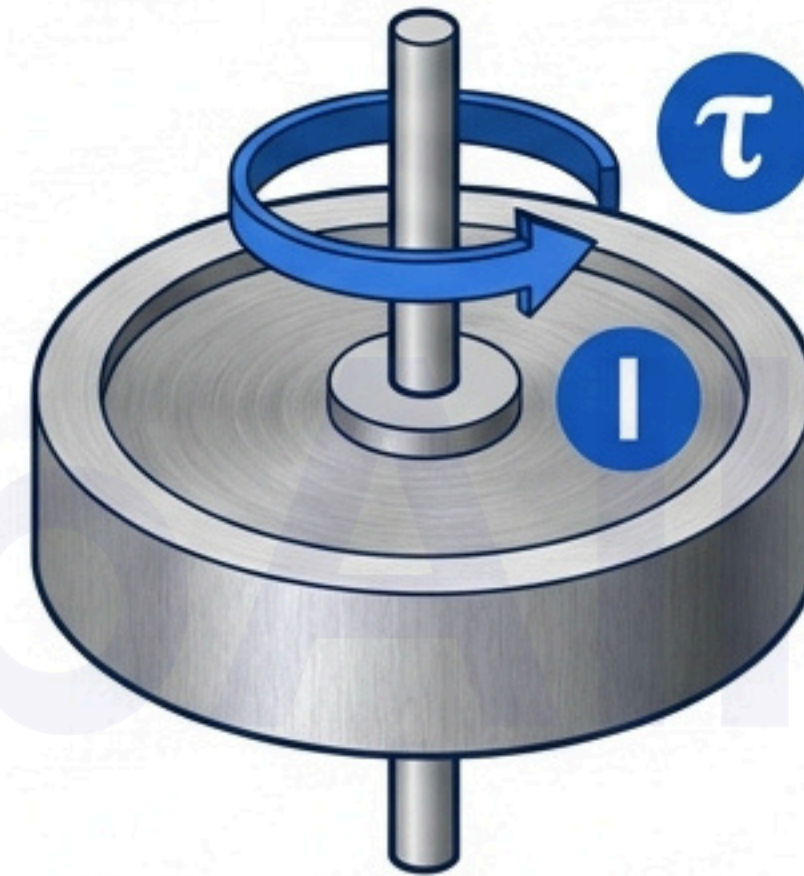
Translation



$$\text{Kinetic Energy: } K = \frac{1}{2} m v^2$$

Functional
Equivalents

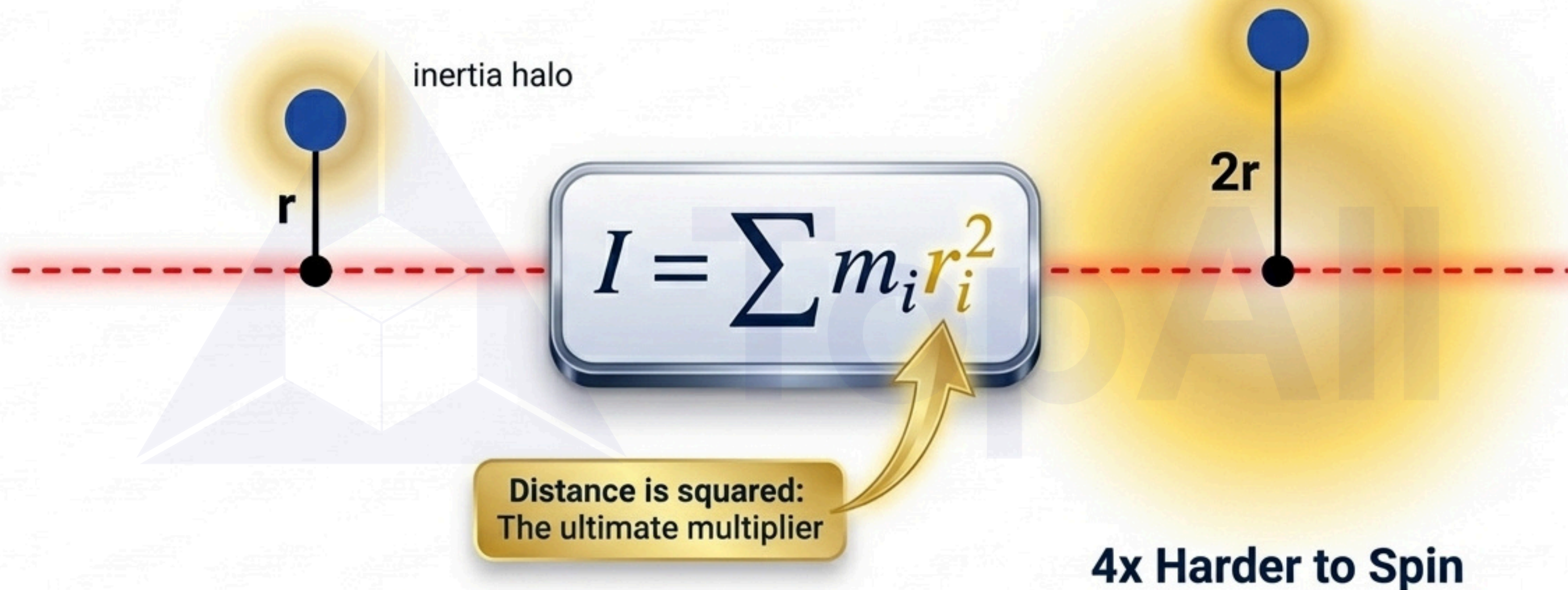
Rotation



$$\text{Kinetic Energy: } K = \frac{1}{2} I \omega^2$$

Mass Distribution Matters

Rotational inertia isn't just about how much mass you have, but **exactly** where it is placed.



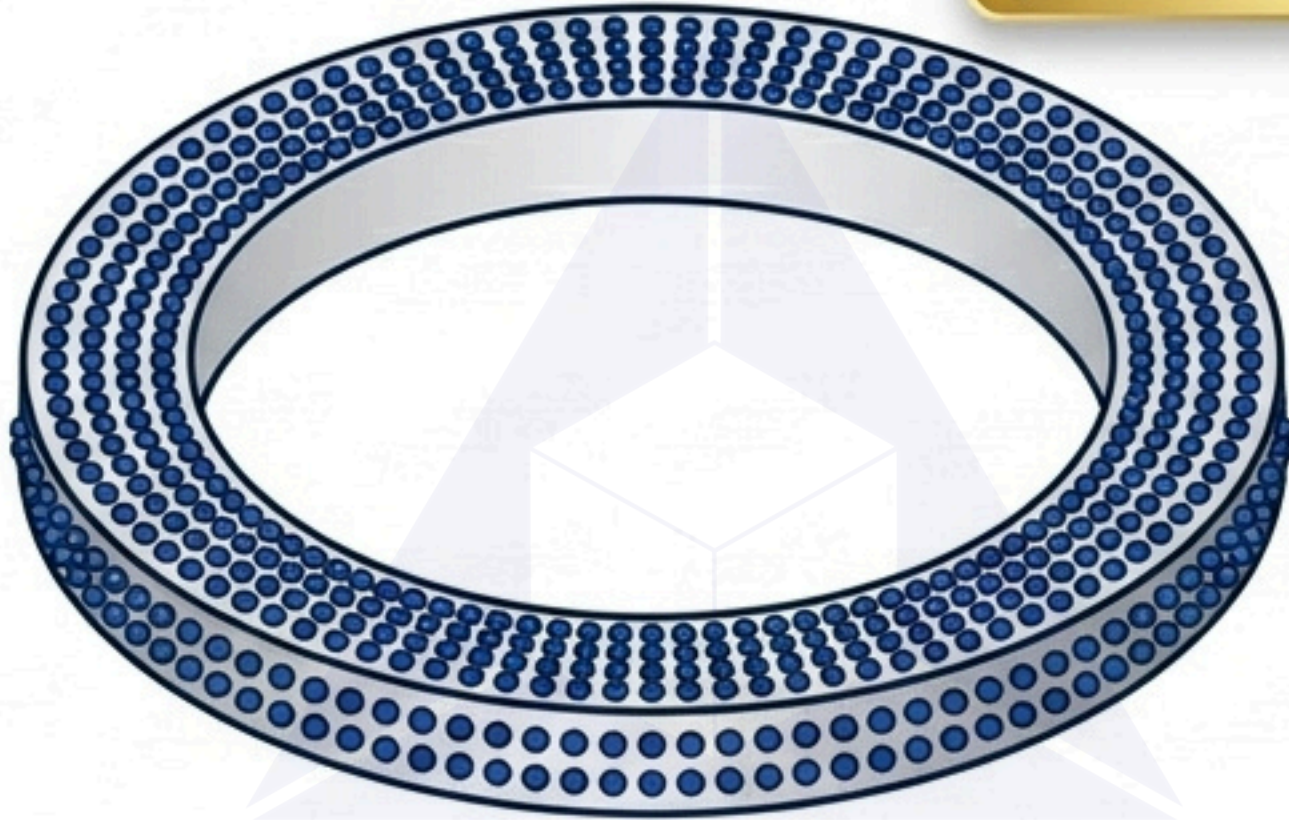
Same Mass, Different Shape

A ring and a disc can weigh the exact same, but the ring is twice as hard to spin.



The Ring

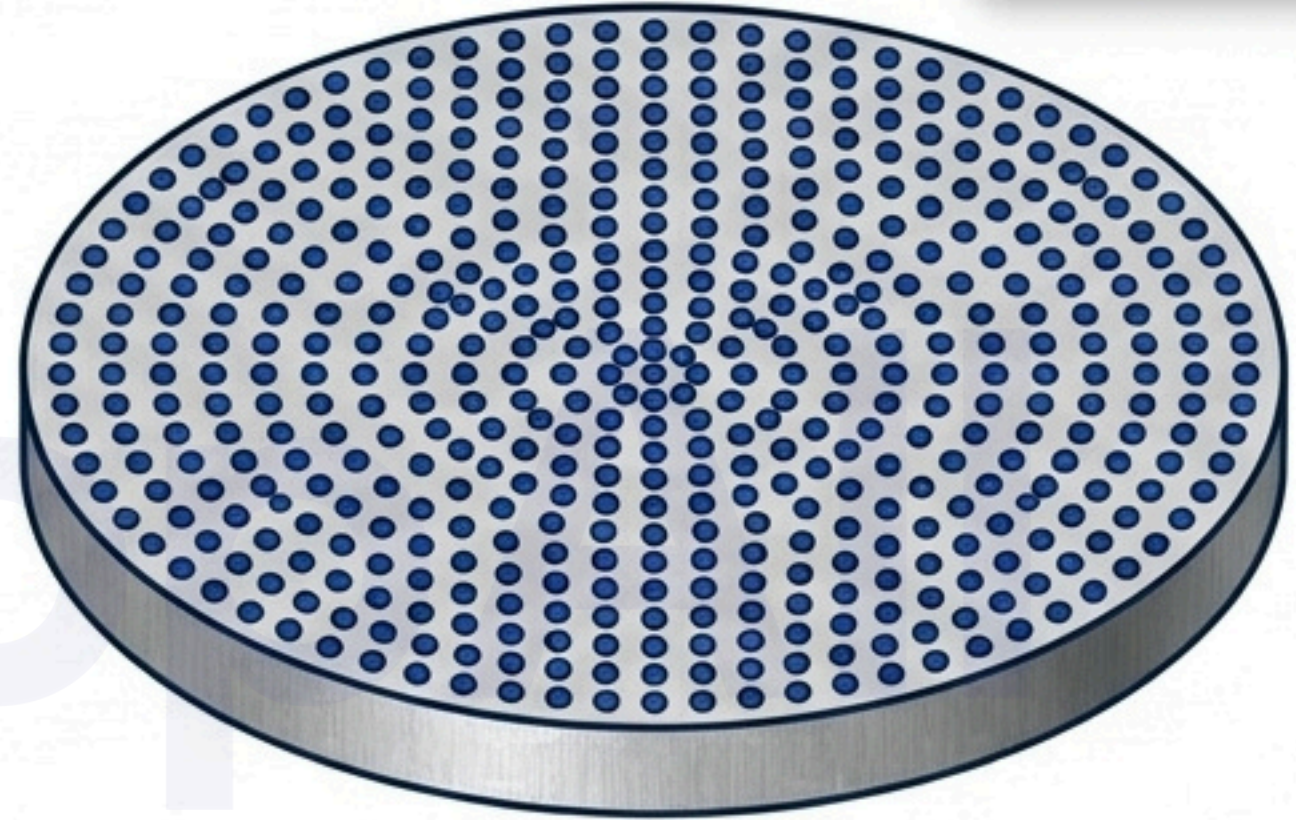
Winner:
Higher Inertia
(Harder to Spin)



$$I = MR^2$$

The Solid Disc

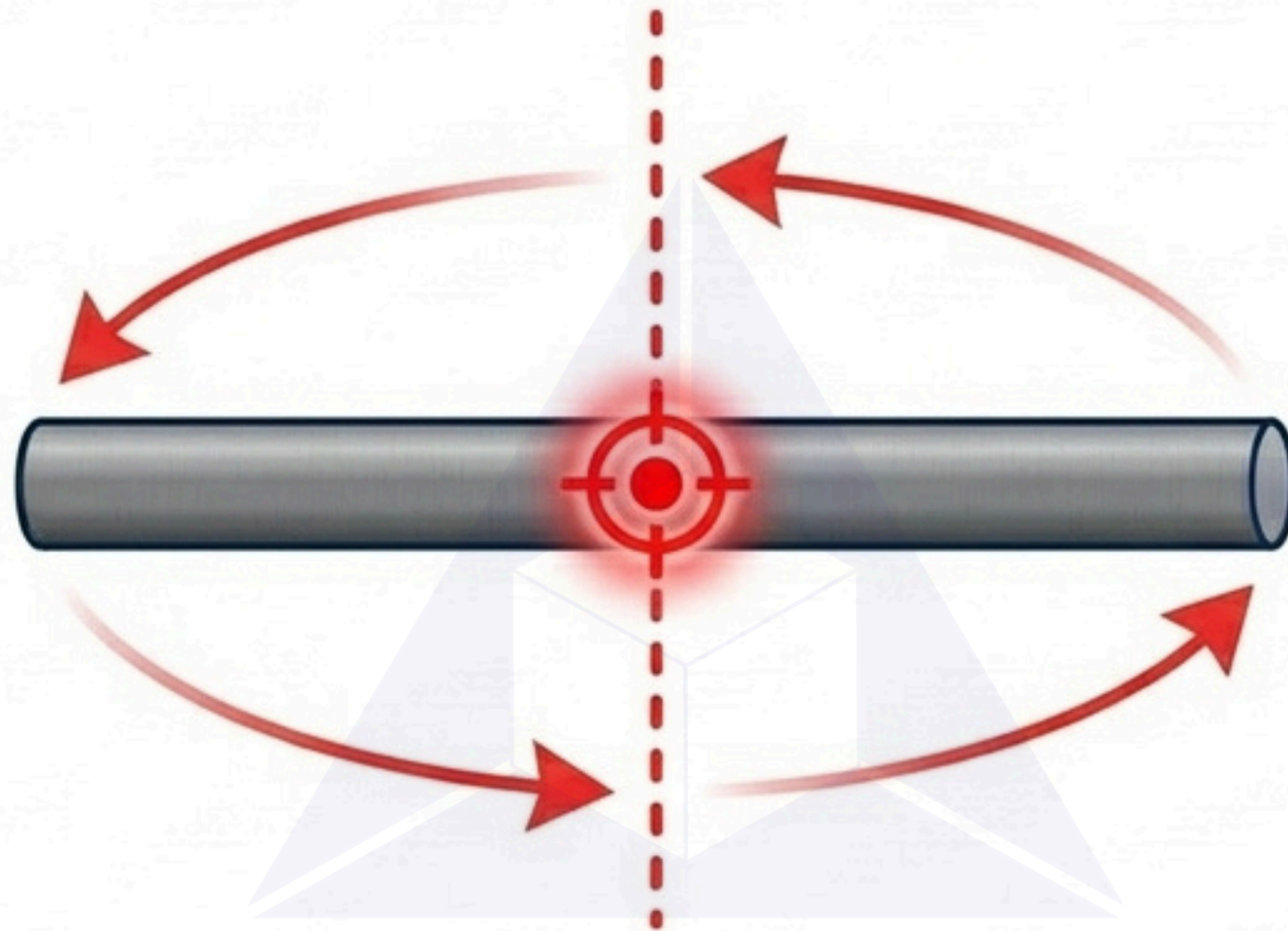
Lower Inertia
(Easier to Spin)



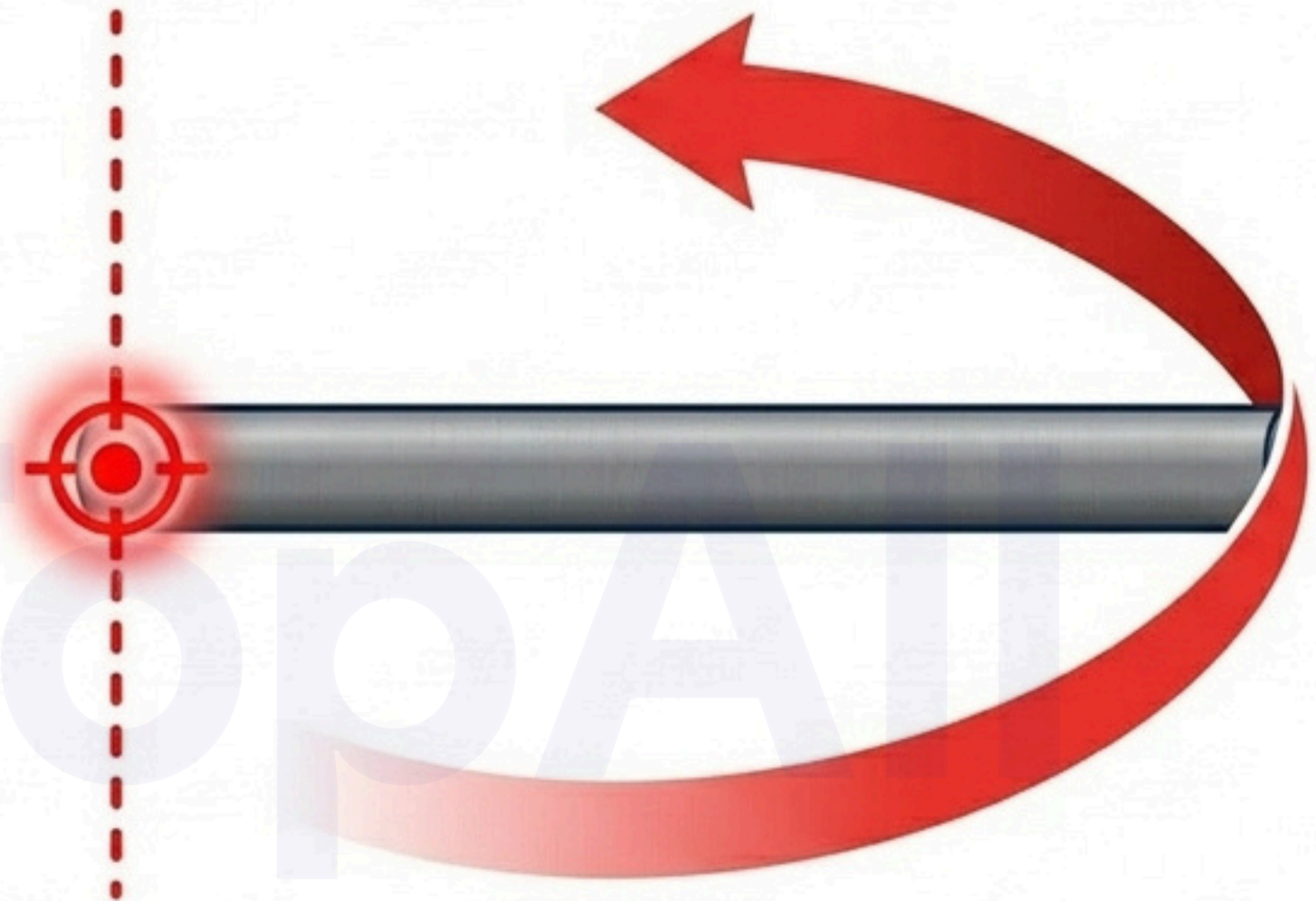
$$I = MR^2/2$$

The Axis Defines the Inertia

Moment of inertia is not a fixed property. Change the pivot point, and you change how mass is distributed relative to the spin.



Balanced distribution.
Lower average r . Easier to spin.



Unbalanced distribution. Bulk of mass
is far from the axis. Harder to spin.

Radius of Gyration Intuition (k)

Imagine crushing a complex object into a single tiny point. Where must you place that point from the axis so the rotational resistance remains exactly the same?



$$I = Mk^2$$

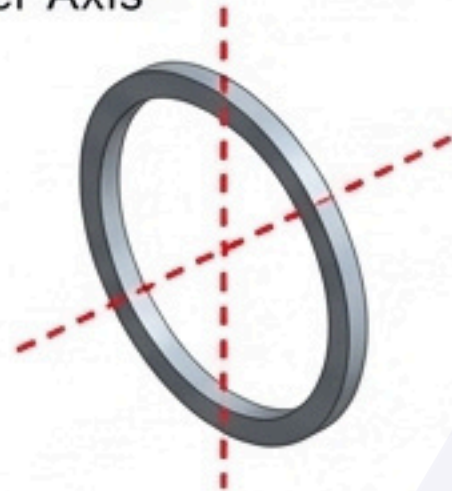
k is a pure geometric property of the body and its axis of rotation.

Standard-Body Values Cheat Sheet

Master reference for uniform rigid bodies.

Thin Circular Ring

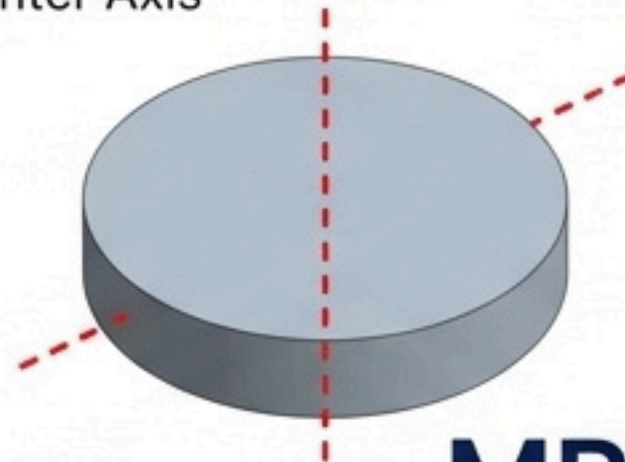
⊕ Center Axis



$$MR^2$$

Circular Disc

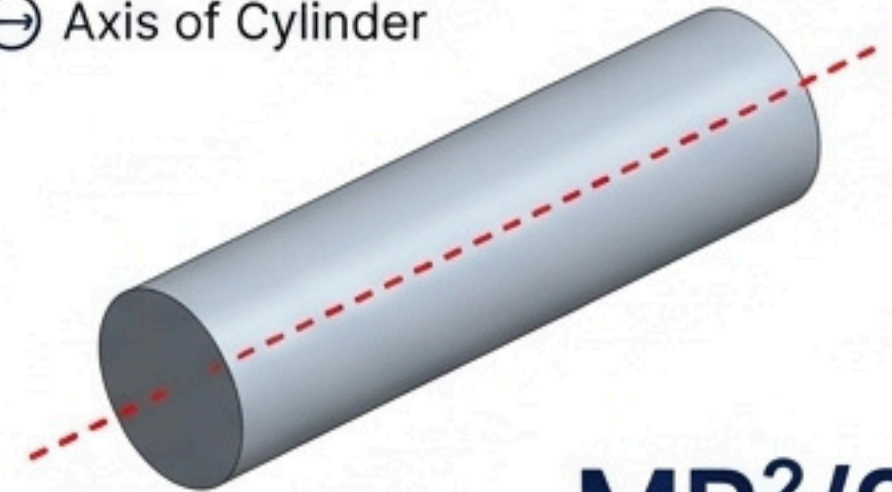
⊕ Center Axis



$$MR^2/2$$

Solid Cylinder

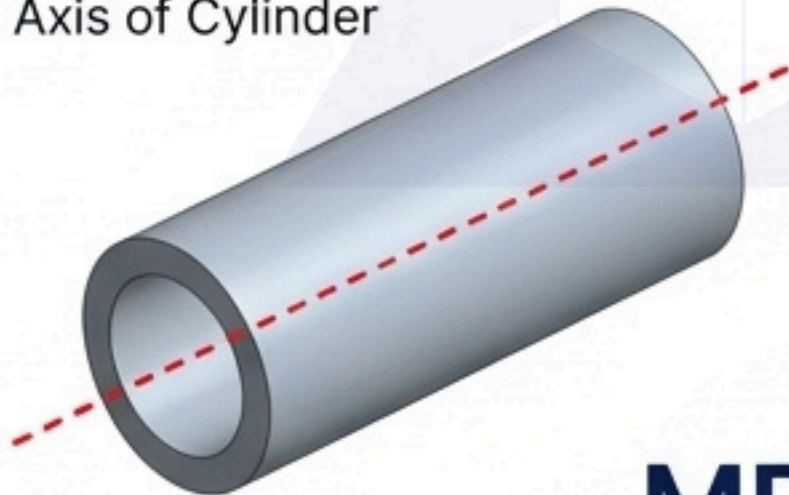
⊖ Axis of Cylinder



$$MR^2/2$$

Hollow Cylinder

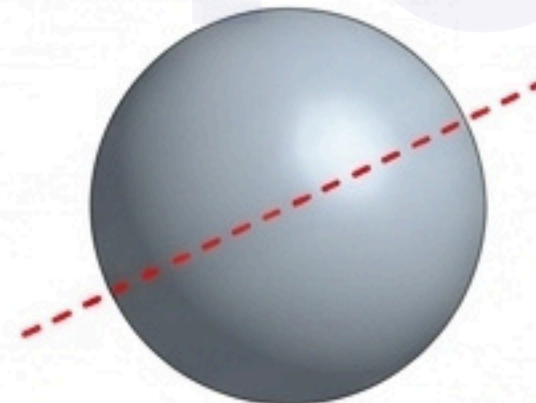
⊖ Axis of Cylinder



$$MR^2$$

Solid Sphere

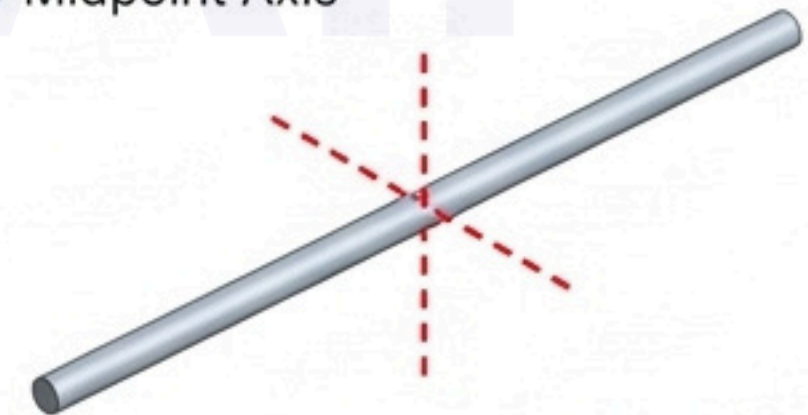
⊖ Diameter Axis



$$2MR^2/5$$

Thin Rod

⊕ Midpoint Axis



$$ML^2/12$$