

The Architecture of Balance

6.8 Equilibrium of a Rigid Body



What Defines Perfect Equilibrium?



Linear Momentum

$$\mathbf{P} = \text{Constant}$$

No linear acceleration ($a = 0$). The center of mass remains at rest or moves with uniform velocity.

Angular Momentum

$$\mathbf{L} = \text{Constant}$$

No angular acceleration ($\alpha = 0$). The body does not start spinning or change its rotational speed.

[Total Mechanical Equilibrium Requires BOTH]

Pillar 1: Translational Equilibrium

[Condition 1]

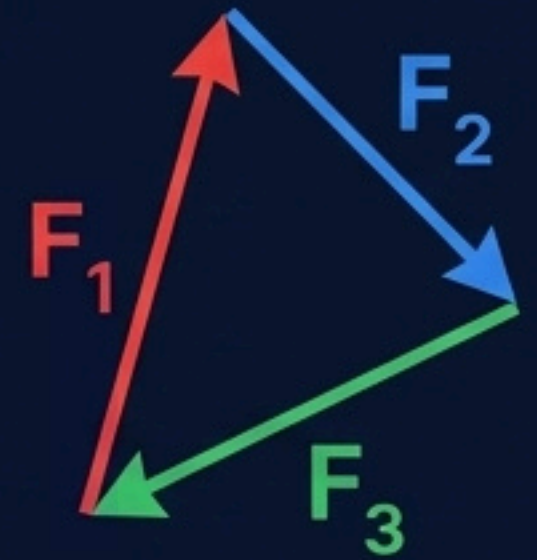
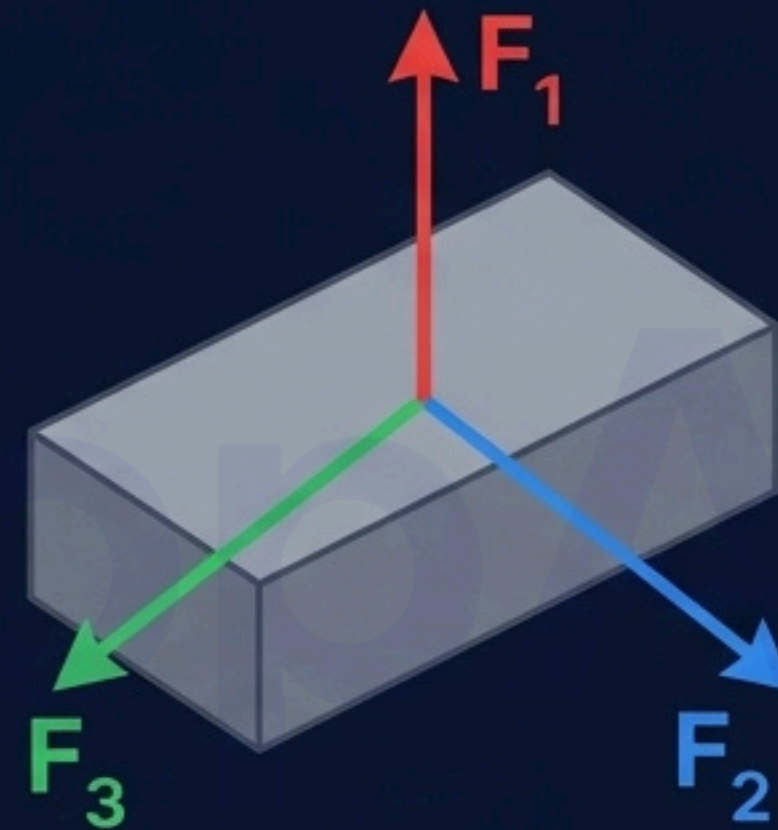
$$\Sigma \mathbf{F} = 0$$

The total external force acting on the rigid body must be zero.

$$F_x = 0$$

$$F_y = 0$$

$$F_z = 0$$



Pillar 2: Rotational Equilibrium

[Condition 2]

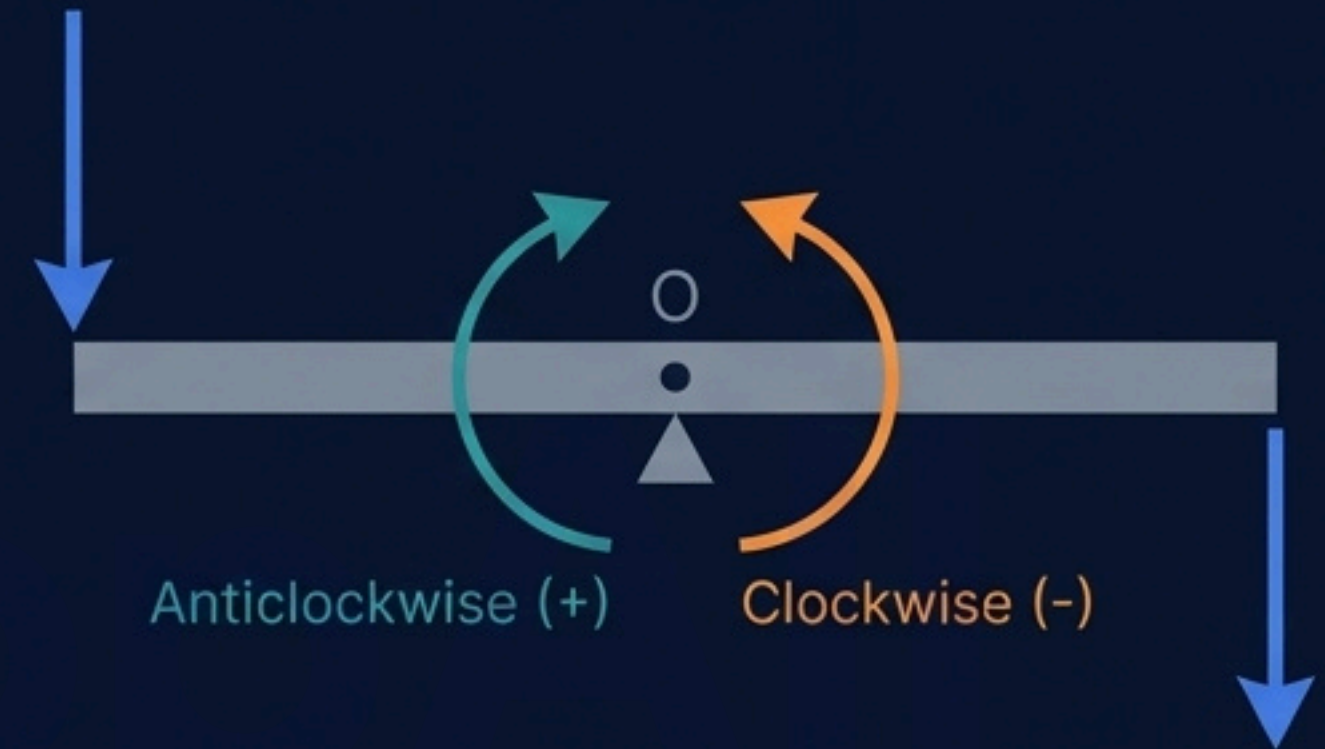
$$\Sigma \tau = 0$$

The total external torque (moment of force) acting on the rigid body must be zero.

$$\tau_x = 0$$

$$\tau_y = 0$$

$$\tau_z = 0$$



Torques perfectly cancel out, regardless of where the origin is chosen.

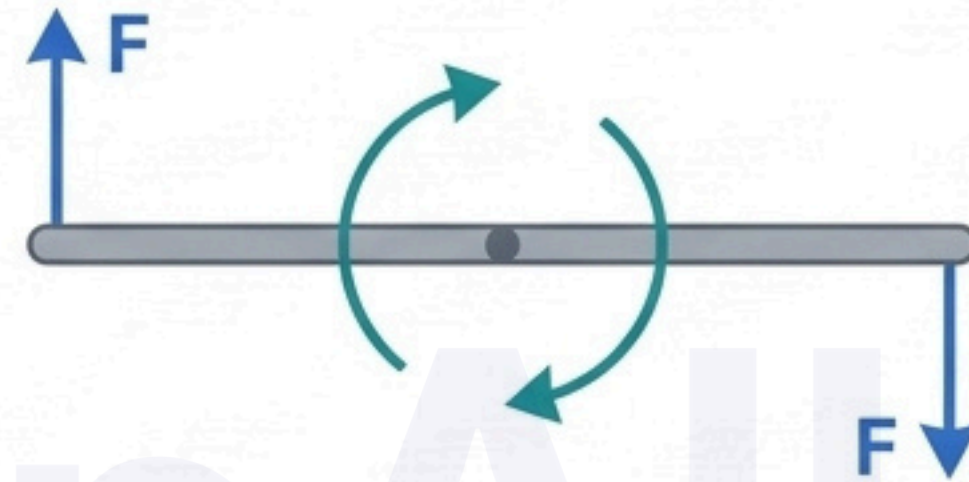
Is One Pillar Enough? (Partial Equilibrium)

Translation without Rotation



- ✗ $\sum F \neq 0$ (Body accelerates upward)
- ✓ $\sum \tau = 0$ (No rotation; forces balance around Center of Mass)

Rotation without Translation

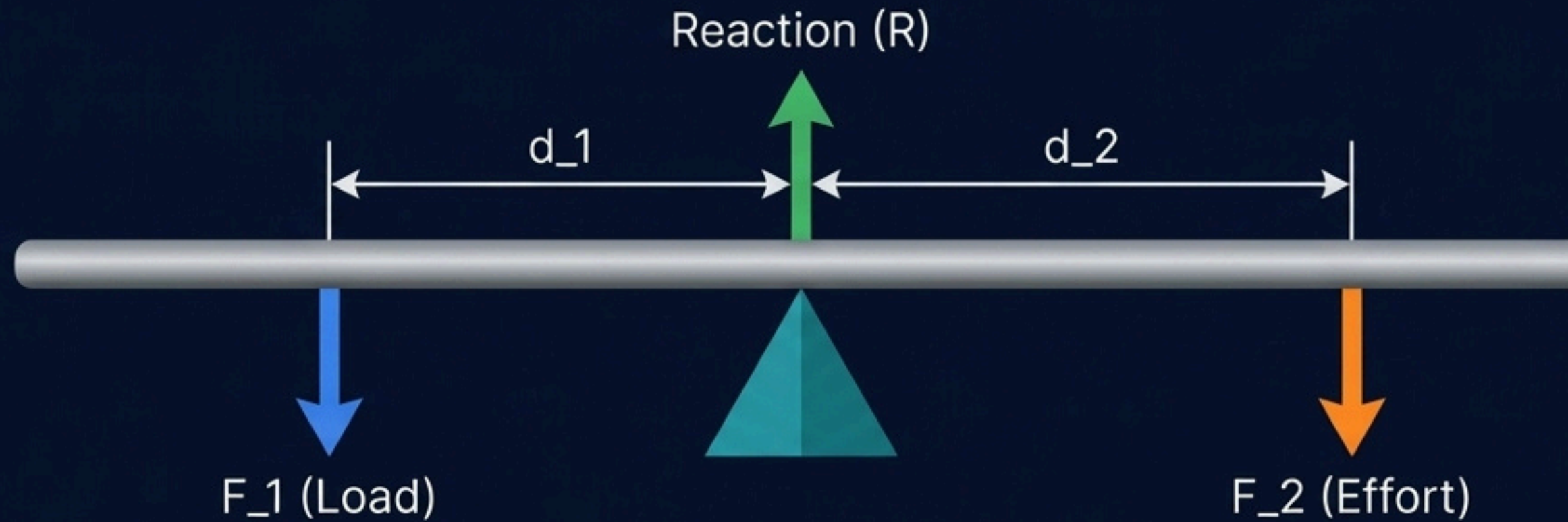


- ✓ $\sum F = 0$ (Forces cancel, center of mass stays put)
- ✗ $\sum \tau \neq 0$ (Body spins!)

[Concept: The Couple]

Equal and opposite forces with different lines of action produce pure rotation.

Perfect Balance in Action: The Lever



Translational Check

Up Forces = Down Forces

$$R - F_1 - F_2 = 0$$

Rotational Check

Principle of Moments

$$d_1 F_1 - d_2 F_2 = 0$$

Load Arm $\times$ Load = Effort Arm $\times$ Effort

The Architecture of Balance: Final Recap



Force Balance (Translational)

Torque Balance (Rotational)

The Core Condition

$$\sum \mathbf{F} = 0$$



$$\sum \boldsymbol{\tau} = 0$$

The Conservation

Momentum $\mathbf{P} = \text{Constant}$



Angular Momentum $\mathbf{L} = \text{Constant}$

The Motion State

Velocity $\mathbf{v} = \text{Constant (or 0)}$



Angular Velocity $\boldsymbol{\omega} = \text{Constant (or 0)}$

The Scalar Components

$$F_x, F_y, F_z = 0$$



$$\tau_x, \tau_y, \tau_z = 0$$

Mechanical Equilibrium is simply the state where the universe agrees to stay perfectly still.