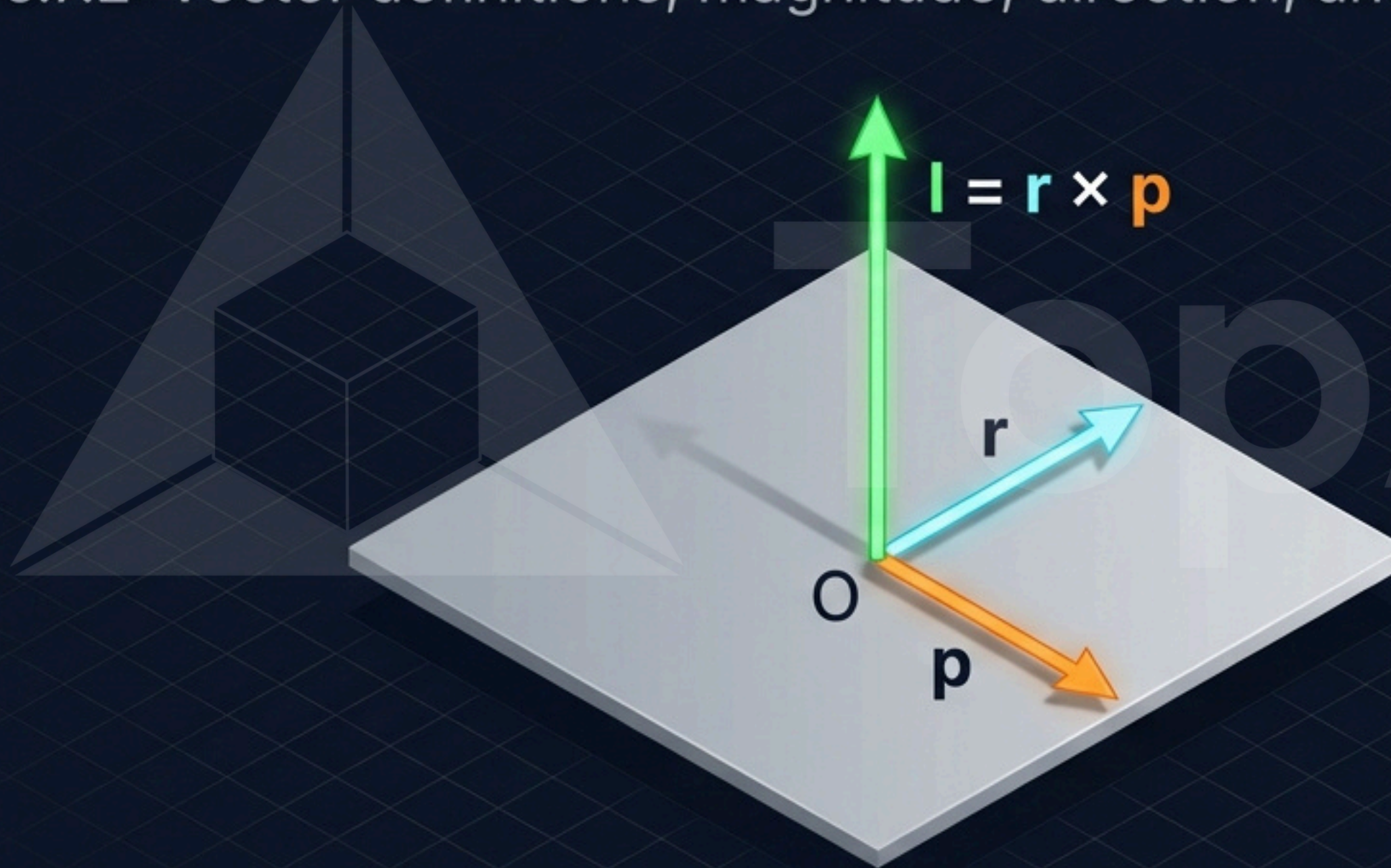


Angular Momentum of a Particle

Section 6.7.2: Vector definitions, magnitude, direction, and origin dependence.

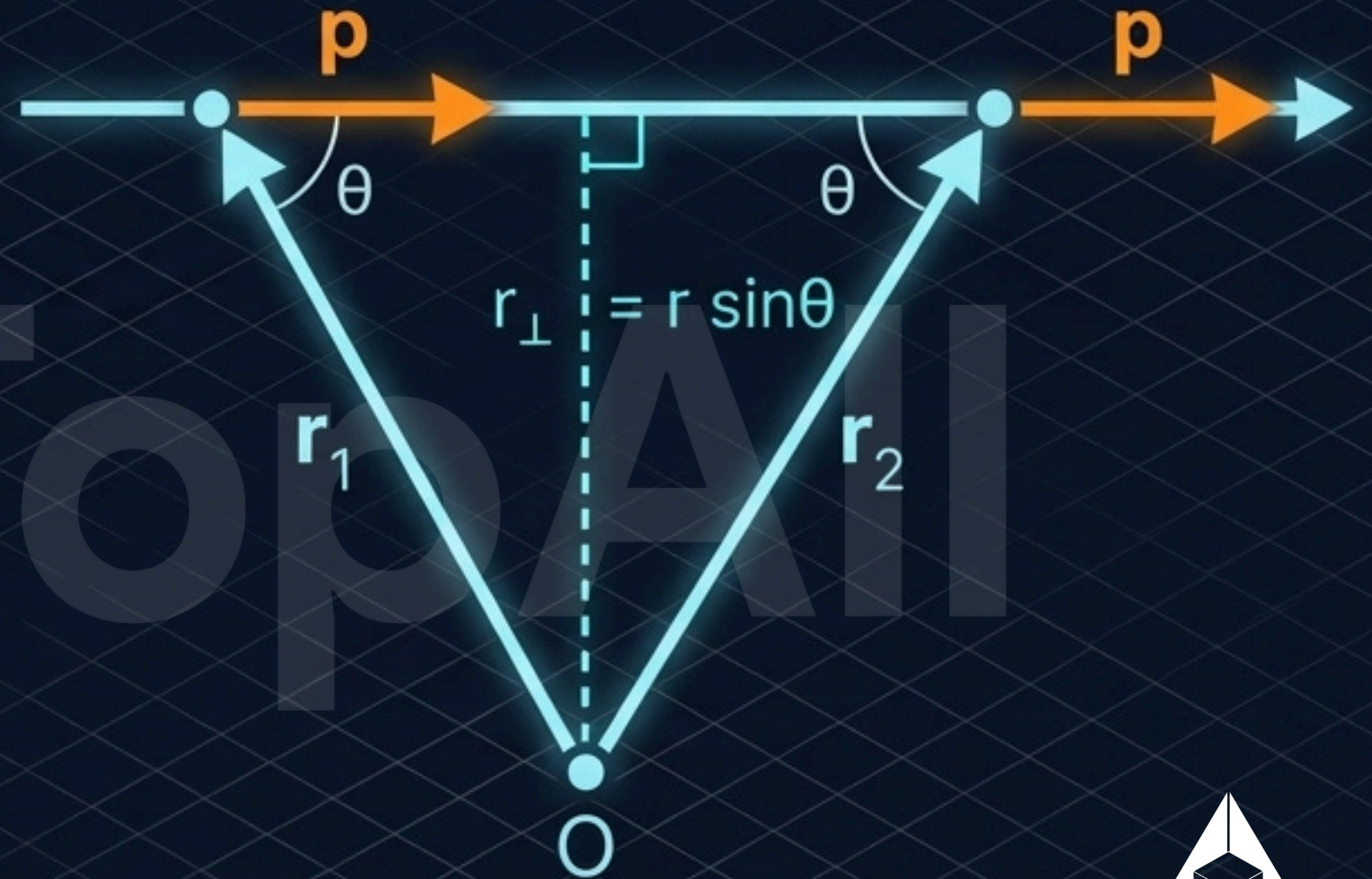


Straight motion can still have angular momentum

A common misconception is that a particle must travel in a circle to have angular momentum.

- Consider a particle moving in a straight line with constant linear momentum ($\mathbf{p}$).
- As it moves, its position vector ($\mathbf{r}$) and angle (θ) relative to origin O change continuously.
- However, the perpendicular distance ($r_{\perp} = r \sin\theta$) remains perfectly constant.

Therefore, its angular momentum remains constant!

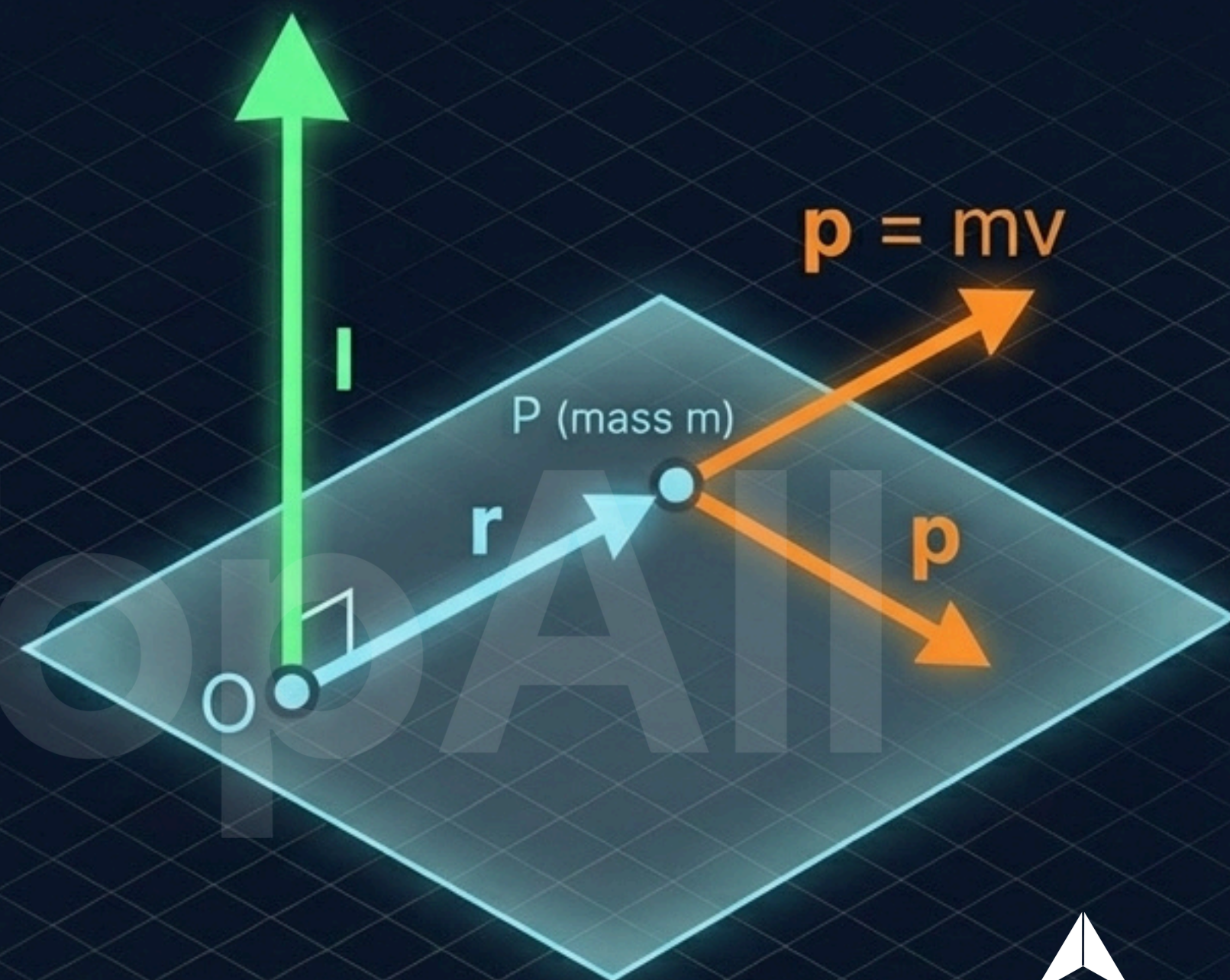


CORE DEFINITION

The Rotational Analogue of Linear Momentum

Angular momentum ($\mathbf{l}$) is the rotational analogue of linear momentum ($\mathbf{p}$). It is defined as the vector product (cross product) of the particle's position vector relative to an origin ($\mathbf{r}$) and its linear momentum ($\mathbf{p}$).

$$\mathbf{l} = \mathbf{r} \times \mathbf{p}$$



Magnitude and Direction

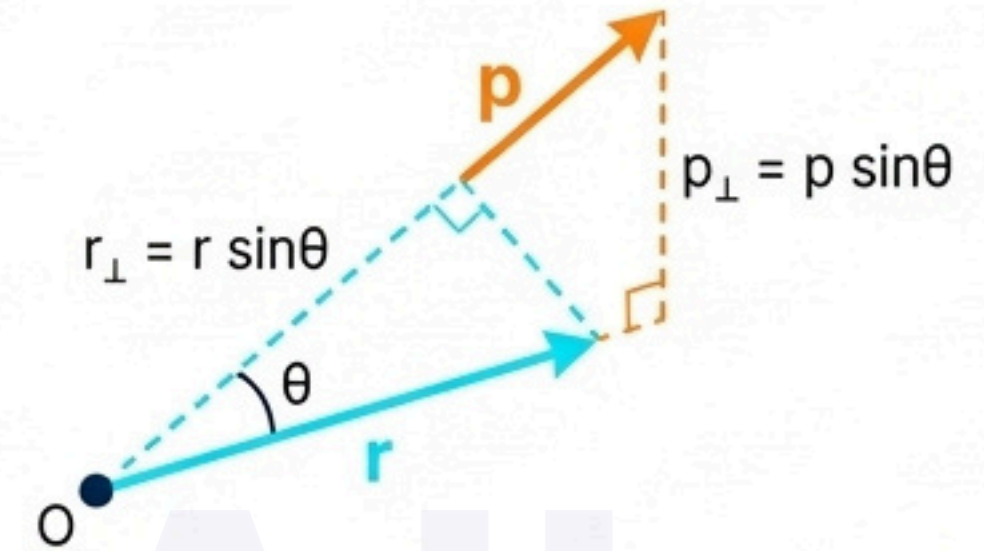
Magnitude

Core Formula: $l = r p \sin\theta$

(where θ is the angle between r and p)

Two Interpretations:

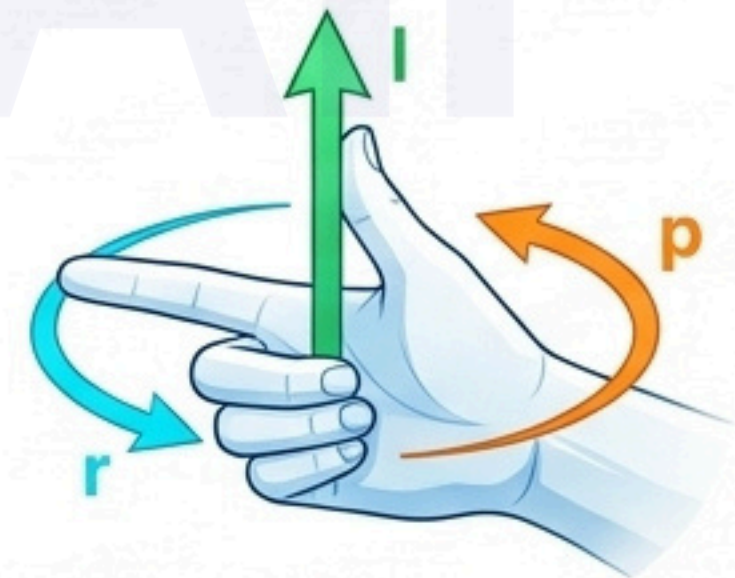
1. $l = p(r \sin\theta) = p r_{\perp}$ (momentum $\times$ perpendicular distance)
2. $l = r(p \sin\theta) = r p_{\perp}$ (position $\times$ perpendicular component of momentum)



Direction

Rule: l is strictly perpendicular to the plane containing both r and p .

Method: Right-Hand Rule. Sweep fingers from r to p ; the thumb points to the direction of l .



Dependence on the Chosen Origin

Angular momentum is not an absolute property; it is defined strictly relative to a specific origin point.

A: Origin on the Line of Action



Math: Angle $\theta = 0^\circ$ (or 180°)

Result: $\sin\theta = 0 \rightarrow l = 0$

Takeaway: If the line of momentum passes through the origin, angular momentum vanishes.

B: Origin off the Line of Action



Math: $r_{\perp} = d > 0$

Result: $l \neq 0$ (Magnitude $l = pd$)

Takeaway: Changing the origin changes both r and θ , creating a non-zero resultant l .

Exam Recap: Particle Angular Momentum

Core Formulas

Vector Definition:

$$\vec{l} = \vec{r} \times \vec{p}$$

Magnitude:

$$l = r p \sin\theta$$

Alternative:

$$\vec{l} = \vec{r}_{\perp} \vec{p} = \vec{r} \vec{p}_{\perp}$$

Zero Conditions ($l = 0$)

- $p = 0$ (Particle is at rest)
- $r = 0$ (Particle is exactly at the origin)
- $\theta = 0^\circ$ or 180° (Line of action of momentum passes directly through the origin)

Relation to Torque

Analogue to Newton's 2nd Law:

$$\frac{d\vec{l}}{dt} = \vec{\tau}$$

The time rate of change of a particle's angular momentum exactly equals the external torque acting on it.