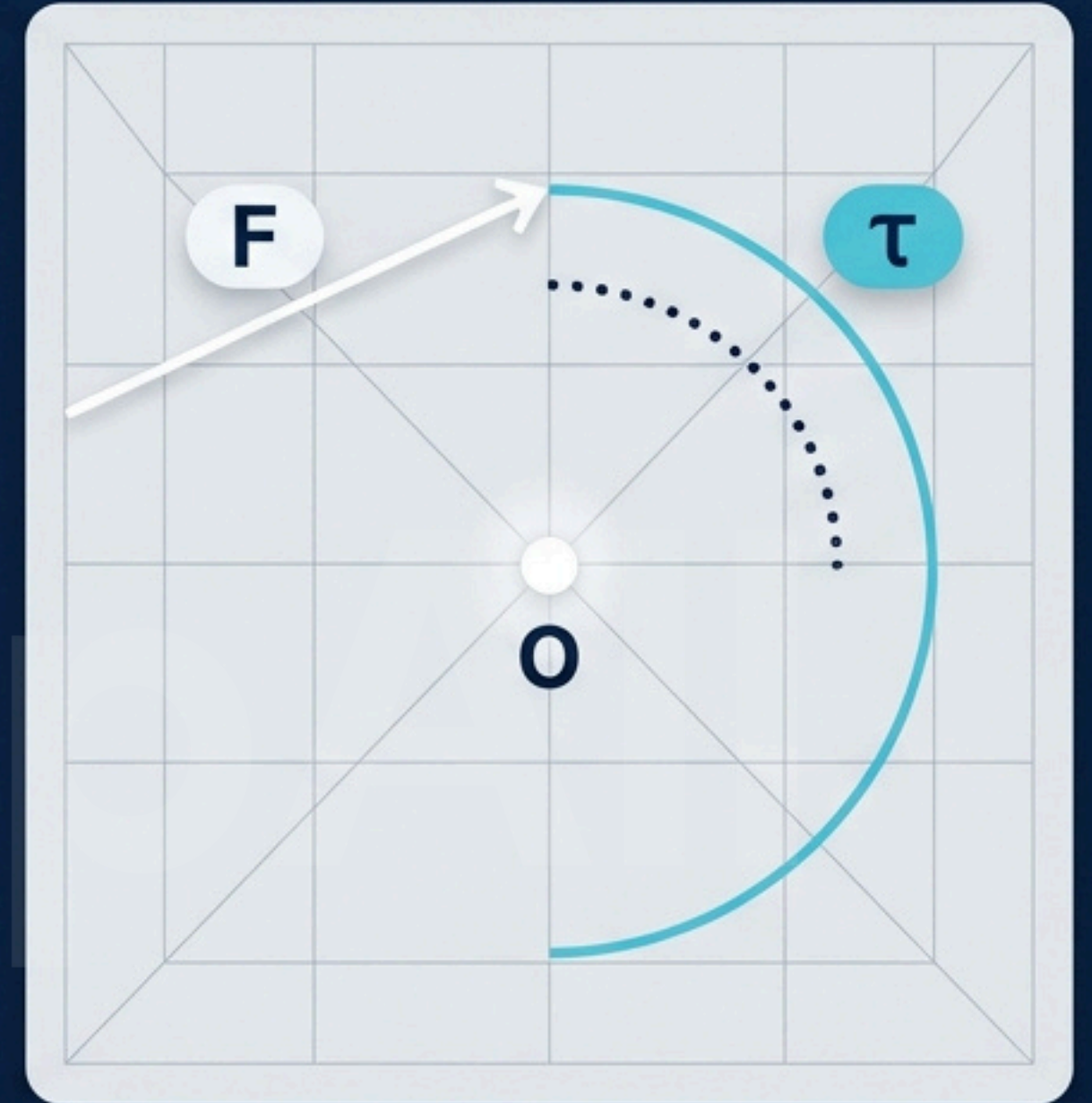


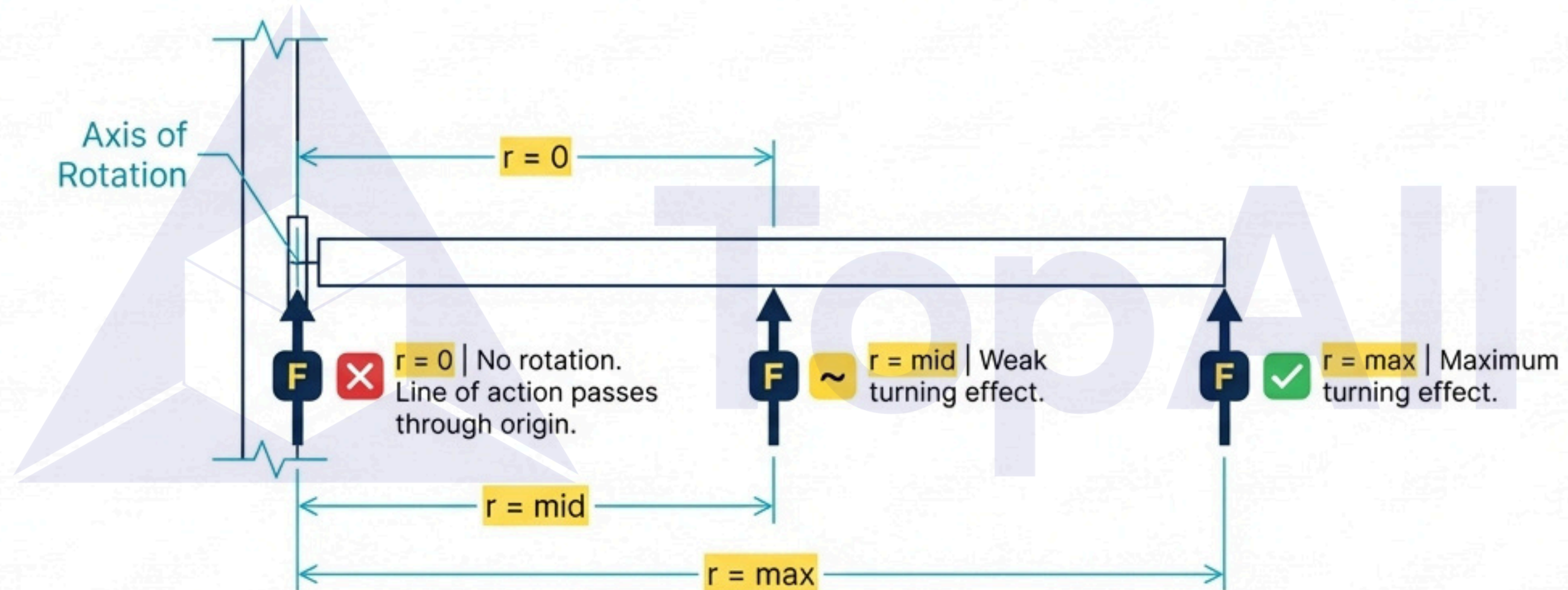
The Rotational Analogue

Bridging Linear Mechanics
to Torque and Angular
Momentum



Why Force Alone is Inadequate

The Effectiveness Gradient



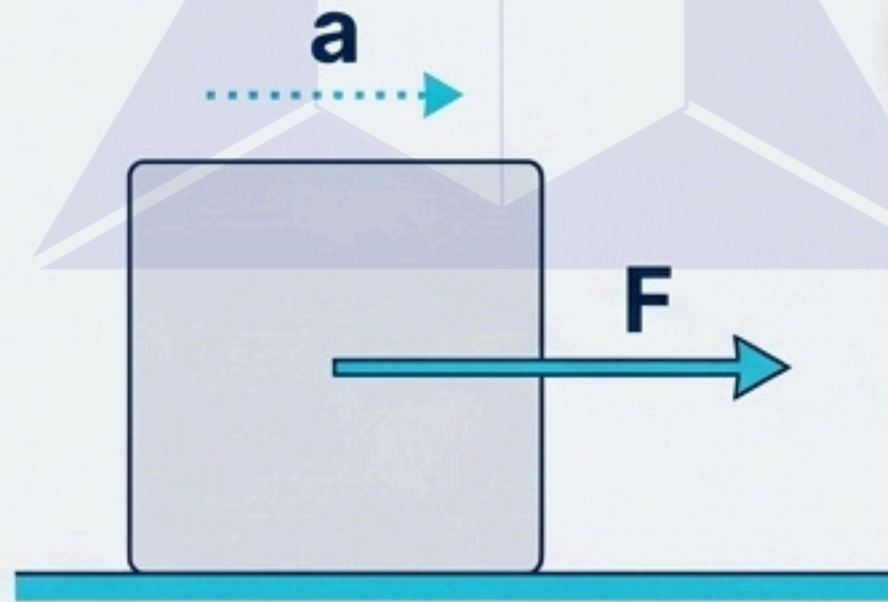
In rotational dynamics, where and how you apply the force (r) is just as critical as the magnitude of the force (F).

Introducing Torque (Moment of Force)

Linear (Translation)

Force ($\mathbf{F}$)

Produces linear acceleration.
Changes the translational state of a body.



Vector Form: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

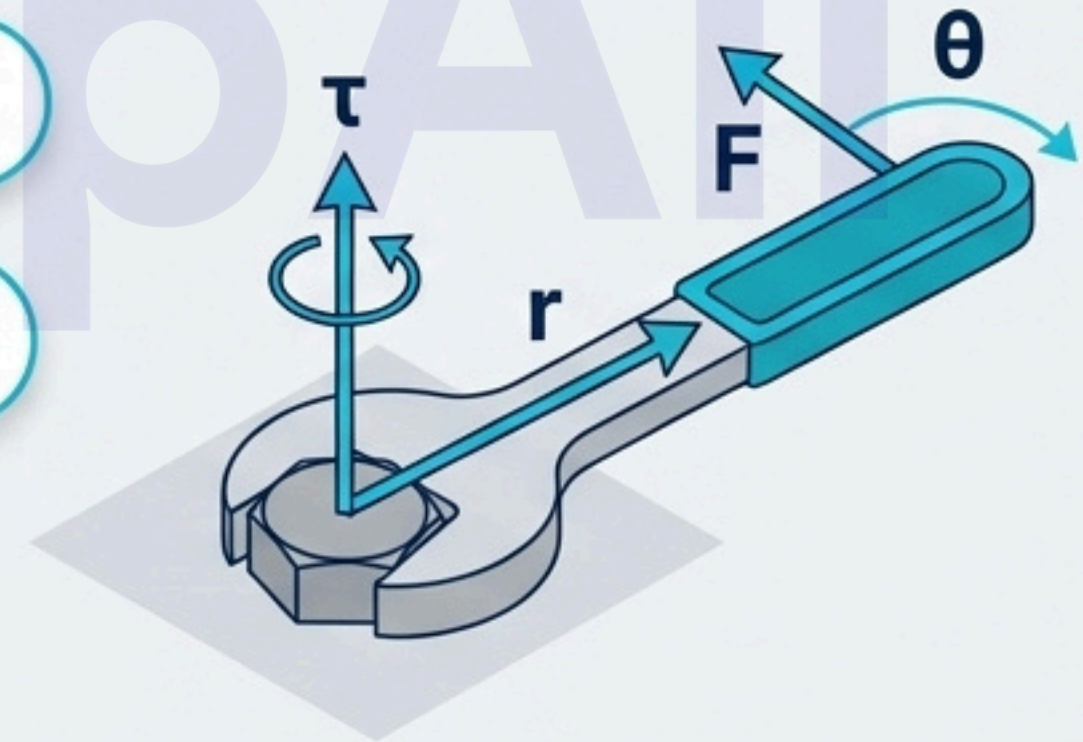
Magnitude: $\tau = rF \sin \theta$

Dimensions: $[ML^2T^{-2}]$

Rotational (Rotation)

Torque ($\boldsymbol{\tau}$)

Produces angular acceleration.
The 'turning effect' of a force.



Scaling the Analogy: Angular Momentum

Linear

Linear Momentum (p)

$$\mathbf{p} = m\mathbf{v}$$

The quantity of linear motion.

Rotational

Angular Momentum ($\mathbf{l}$)

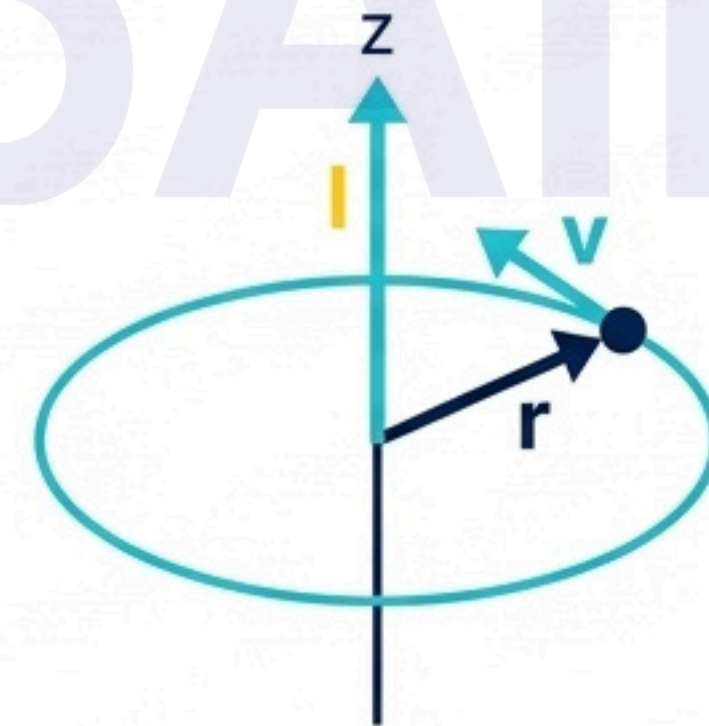
$$\mathbf{l} = \mathbf{r} \times \mathbf{p}$$

The moment of linear momentum.

Newton's Second Law mapped to rotation:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \longrightarrow \boldsymbol{\tau} = \frac{d\mathbf{l}}{dt}$$

The time rate of change of angular momentum equals the external torque.



The Rosetta Stone of Mechanics

Direct mapping of translational variables to rotational variables.



Concept	Translational Motion	Rotational (Fixed Axis)
Inertia	Mass (M)	Moment of Inertia (I)
Kinematics	Velocity ($\mathbf{v} = dx/dt$)	Angular Velocity ($\boldsymbol{\omega} = d\theta/dt$)
Kinematics	Acceleration ($\mathbf{a} = dv/dt$)	Angular Accel. ($\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$)
Dynamics	Force ($\mathbf{F}$)	Torque ($\boldsymbol{\tau}$)
Dynamics	Momentum ($\mathbf{p} = M\mathbf{v}$)	Angular Momentum ($\mathbf{L} = I\boldsymbol{\omega}$)
Newton's 2nd Law	$\mathbf{F} = Ma$	$\boldsymbol{\tau} = I\boldsymbol{\alpha}$

Core Takeaways & Next Steps



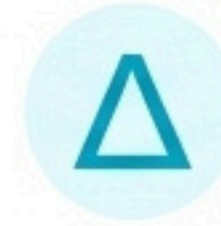
The Cross Product Rule

Both τ and L are vector products ($\mathbf{r} \times \mathbf{F}$ and $\mathbf{r} \times \mathbf{p}$). Direction is always determined by the right-hand screw rule. Order matters: $\mathbf{r}$ comes first!



The Axis is Everything

r is always the perpendicular distance from the specific axis of rotation. If the line of action passes through the origin ($r = 0$ or $\theta = 0^\circ$), the rotational effect is strictly zero.



The Ultimate Dynamic

If net external torque is zero ($\tau = 0$), angular momentum is conserved ($dL/dt = 0$). This governs spinning tops, planets, and acrobats.

Next Module: Kinematics of Rotational Motion →